

# Charm Physics at BESIII

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**For BESIII Collaboration**

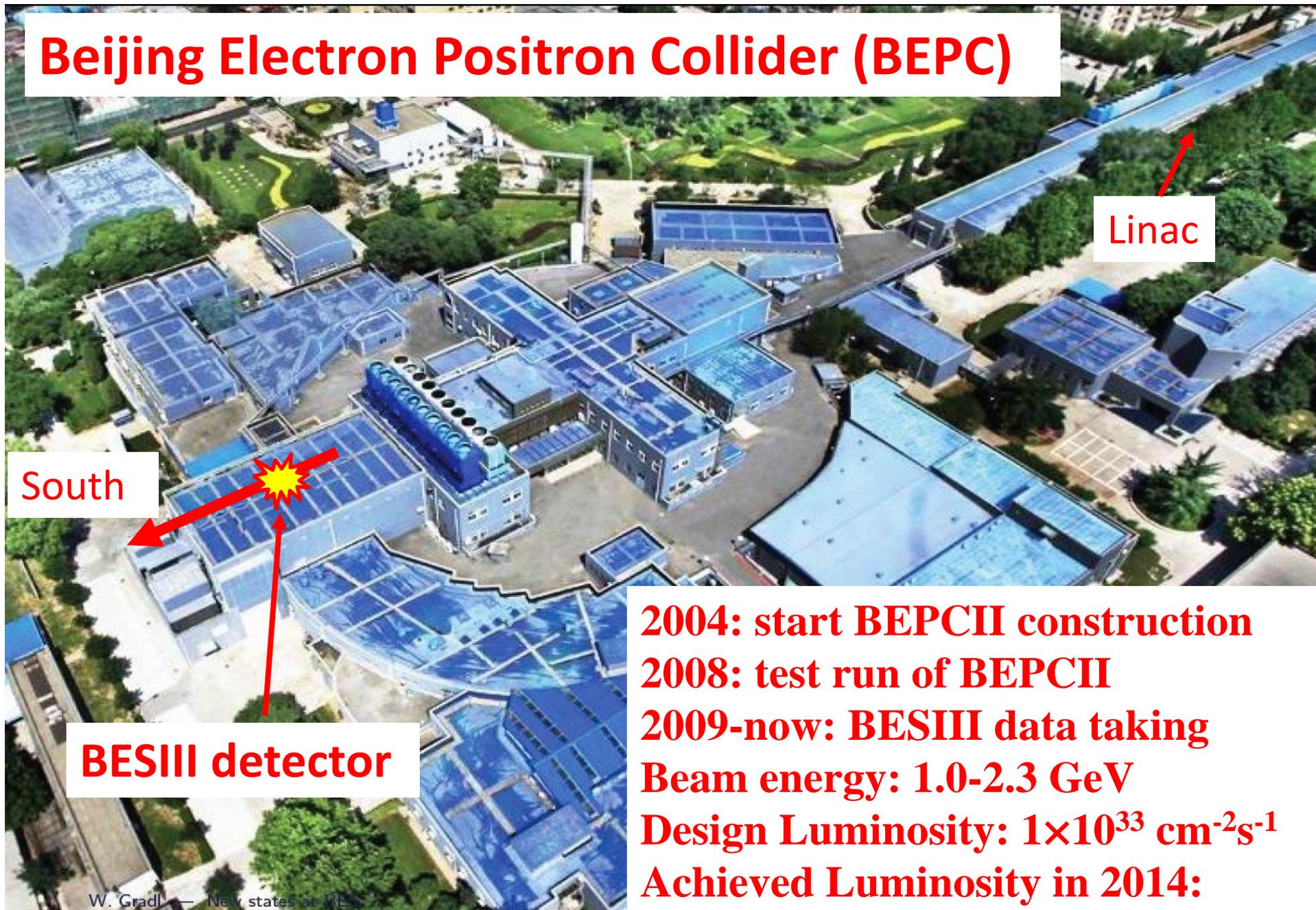
**Beijing Institute of Petro-chemical Technology (BIPT)**

**54<sup>th</sup> International Winter Meeting On Nuclear Physics  
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# Outline

- **Introduction**
- **D decays**
  - **D leptonic and semi-leptonic decays**
  - **D hadronic decays**
- **$\Lambda_c^+$  decays**
  - **$\Lambda_c^+$  semi-leptonic decays**
  - **$\Lambda_c^+$  hadronic decays**
- **Summary**

# Beijing Electron Positron Collider (BEPC)



Linac

South

BESIII detector

**2004: start BEPCII construction**  
**2008: test run of BEPCII**  
**2009-now: BESIII data taking**  
**Beam energy: 1.0-2.3 GeV**  
**Design Luminosity:  $1 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$**   
**Achieved Luminosity in 2014:**

$$L_{\text{peak}} = 0.85 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$$

# BESIII Detector

## Drift Chamber (MDC)

$$\sigma_{P/P} (\%) = 0.5\% (1\text{GeV})$$

$$\sigma_{dE/dx} (\%) = 6\%$$

## Time Of Flight (TOF)

$$\sigma_T: 90 \text{ ps Barrel}$$

$$110 \text{ ps endcap}$$

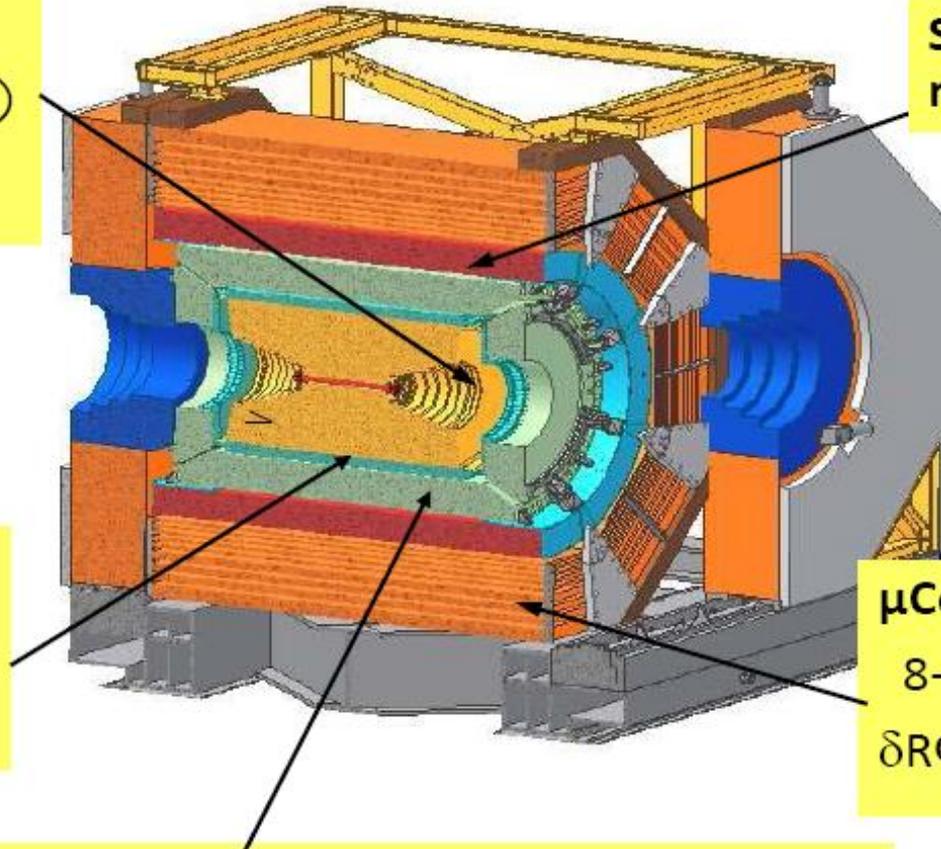
**EMC:**  $\sigma_{E/\sqrt{E}} (\%) = 2.5\% (1 \text{ GeV})$   
(CsI)  $\sigma_{z,\phi} (\text{cm}) = 0.5 - 0.7 \text{ cm}/\sqrt{E}$

Super-conducting  
magnet (1.0 tesla)

## $\mu$ Counter

8- 9 layers RPC

$$\delta R\Phi = 1.4 \text{ cm} \sim 1.7 \text{ cm}$$

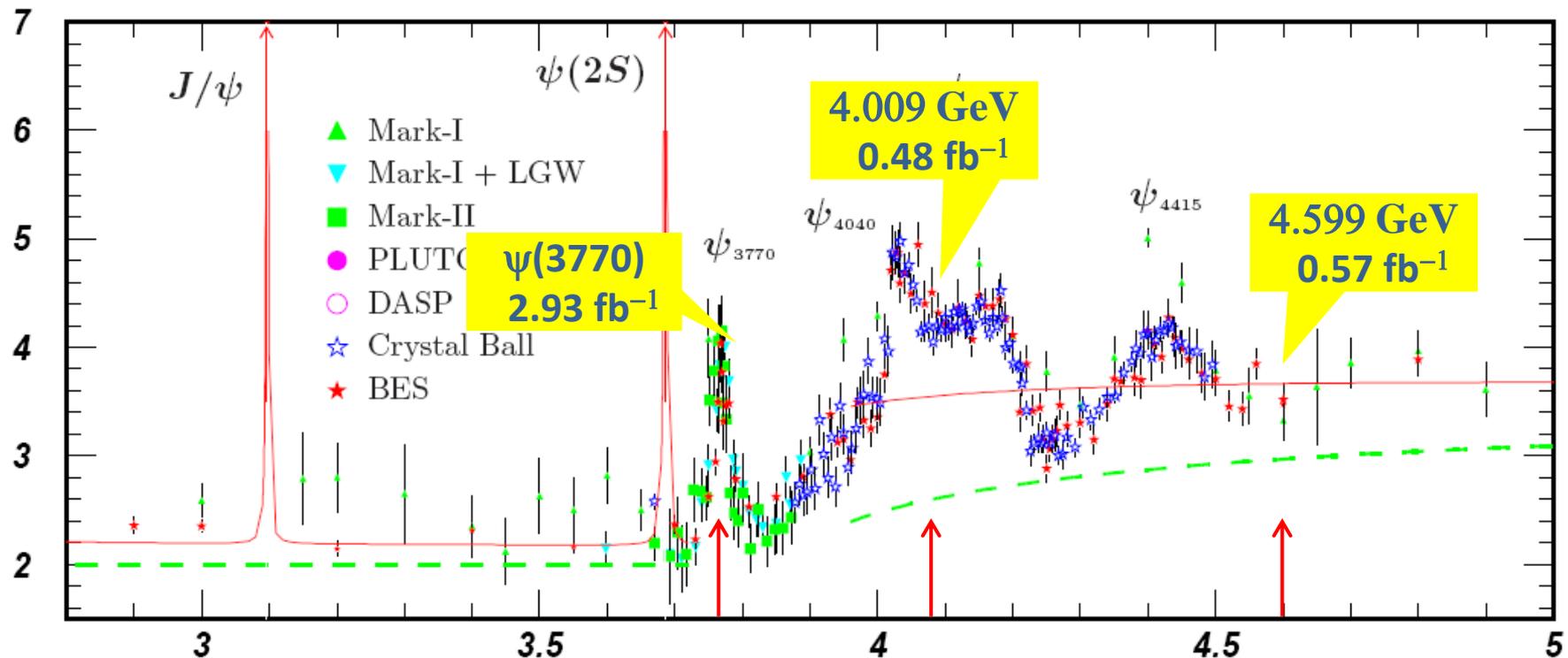


# Data samples in this talk

□ 2.93 fb<sup>-1</sup> data@3.773 GeV for D<sup>0</sup> $\bar{D}^0$ , D<sup>+</sup>D<sup>-</sup> production;

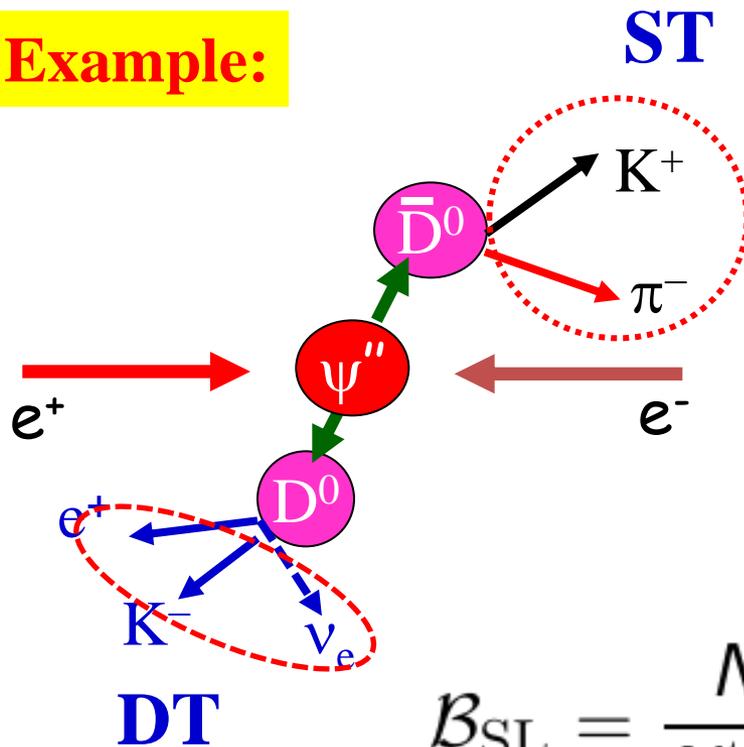
□ 0.48 fb<sup>-1</sup> data@4.009 GeV for D<sub>s</sub><sup>+</sup>D<sub>s</sub><sup>-</sup> production;

□ 0.57 fb<sup>-1</sup> data@4.599 GeV for  $\Lambda_C^+$   $\bar{\Lambda}_C^-$  production;



# Analysis Technique

**Example:**



Single Tags (ST)

$$M_{bc} = \sqrt{E_{\text{beam}}^2 - |\vec{p}_{\bar{D}^0}|^2}$$

Double Tags (ST)

$$U_{\text{miss}} = E_{\text{miss}} - c|\vec{p}_{\text{miss}}|$$

[For Semi-Leptonic decays]

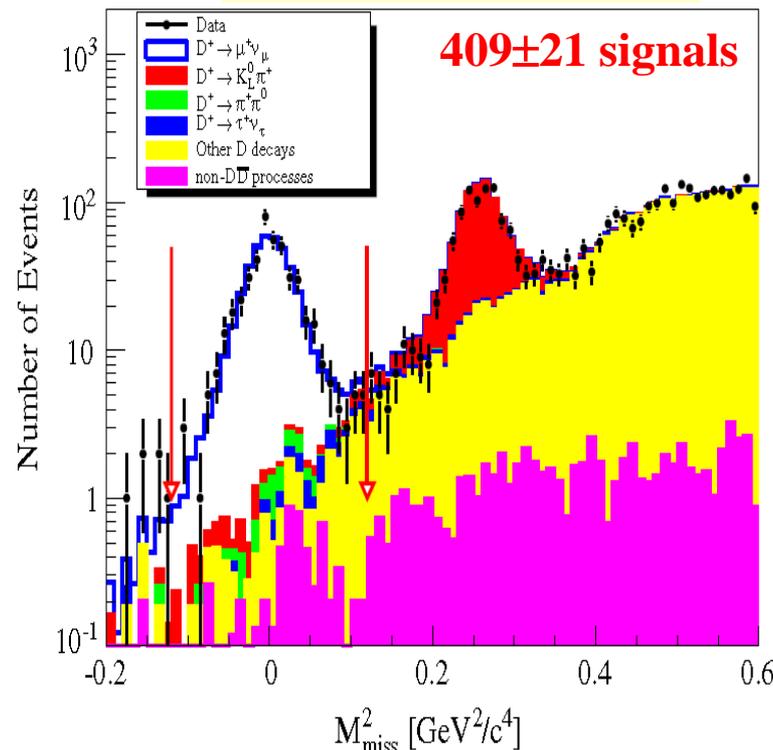
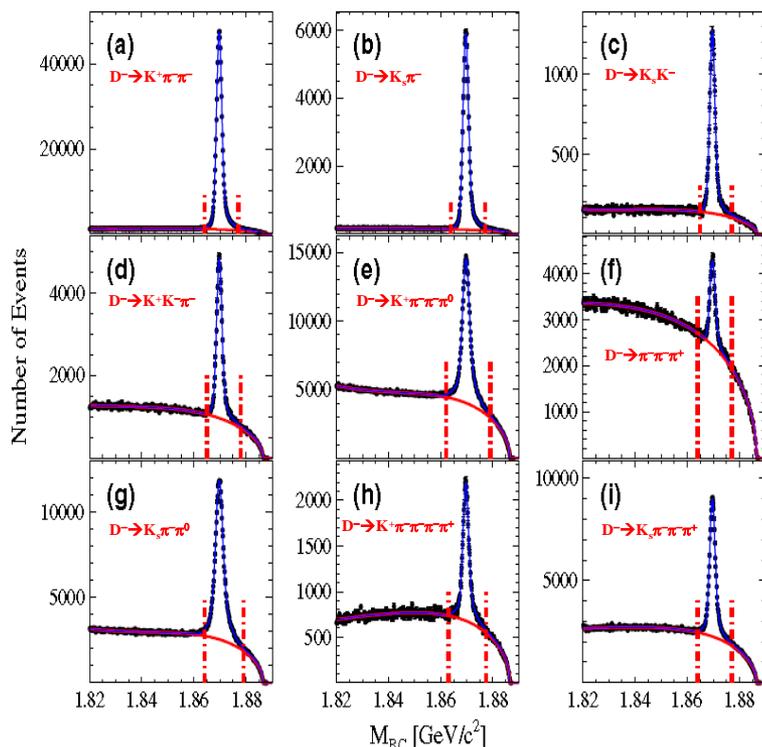
$$\mathcal{B}_{\text{SL}} = \frac{N^{\text{semi}}}{N^{\text{tag}} \times \epsilon}$$

**Clean sample of ST charmed hadrons can be fully reconstructed by hadronic decays with large BFs. Based on this, one can access to absolute BFs and dynamics in the decays.**

# Measurement of $B[D^+ \rightarrow \mu^+ \nu]$ , $f_{D^+}$ and $|V_{cd}|$

$$e^+e^- \rightarrow \psi(3770) \rightarrow D^+D^-$$

2.93 fb<sup>-1</sup> data@ 3.773 GeV  
PRD89(2014)051104R



$$N_{D_{\text{tag}}} = (170.31 \pm 0.34) \times 10^4$$

$$B[D^+ \rightarrow \mu^+ \nu] = (3.71 \pm 0.19 \pm 0.06) \times 10^{-4}$$

Input  $t_{D^+}$ ,  $m_{D^+}$ ,  $m_{\mu^+}$  on PDG  
and  $|V_{cd}|$  of CKM-Fitter

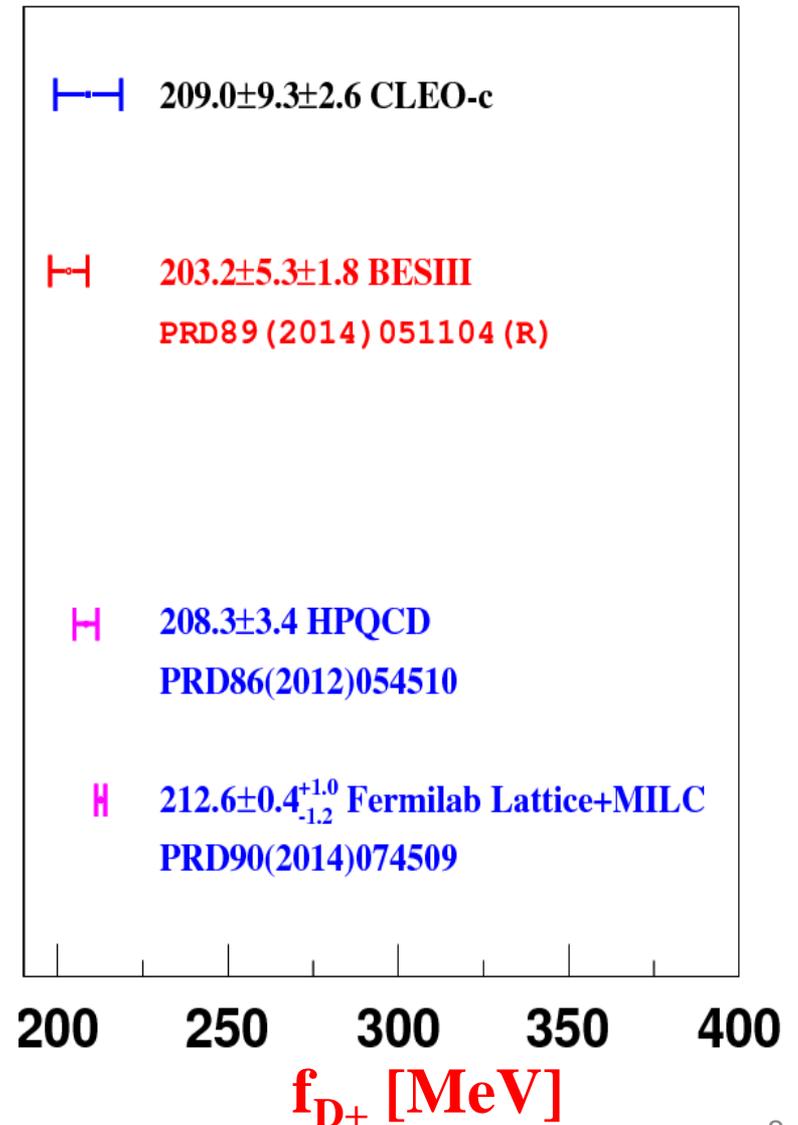
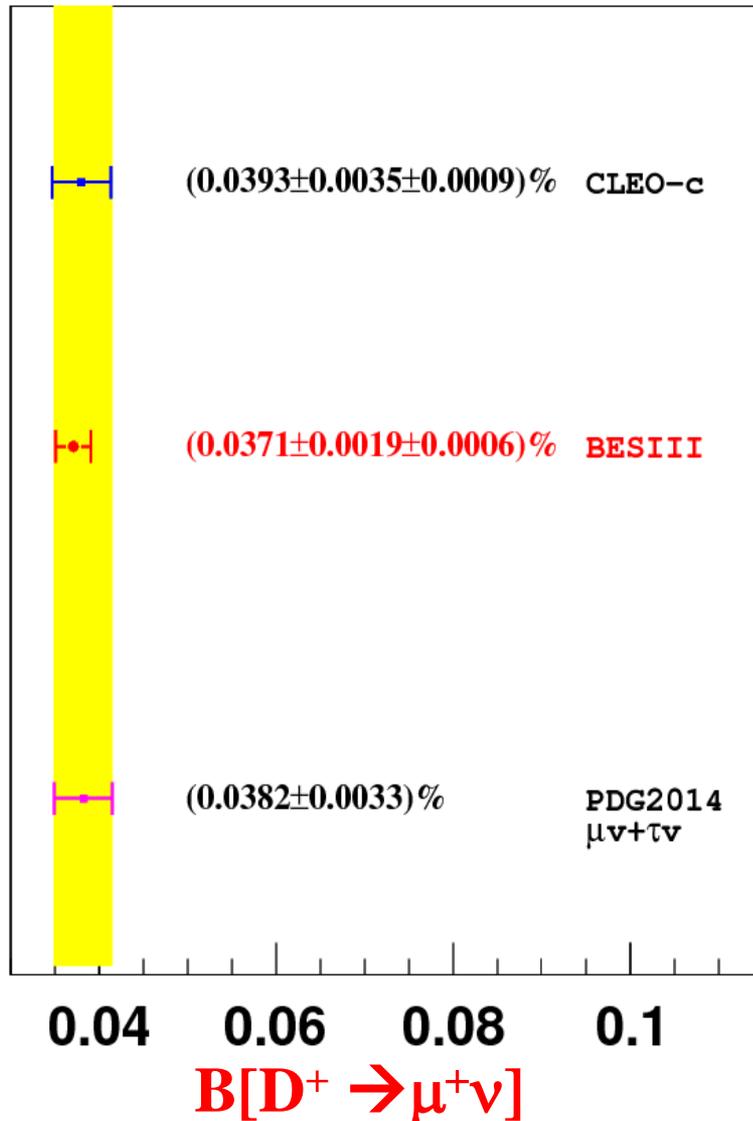
BES III

Input  $t_{D^+}$ ,  $m_{D^+}$ ,  $m_{\mu^+}$  on PDG and  
LQCD calculated  $f_{D^+} = 207 \pm 4$   
MeV [PRL100(2008)062002]

$$f_{D^+} = (203.2 \pm 5.3 \pm 1.8) \text{ MeV}$$

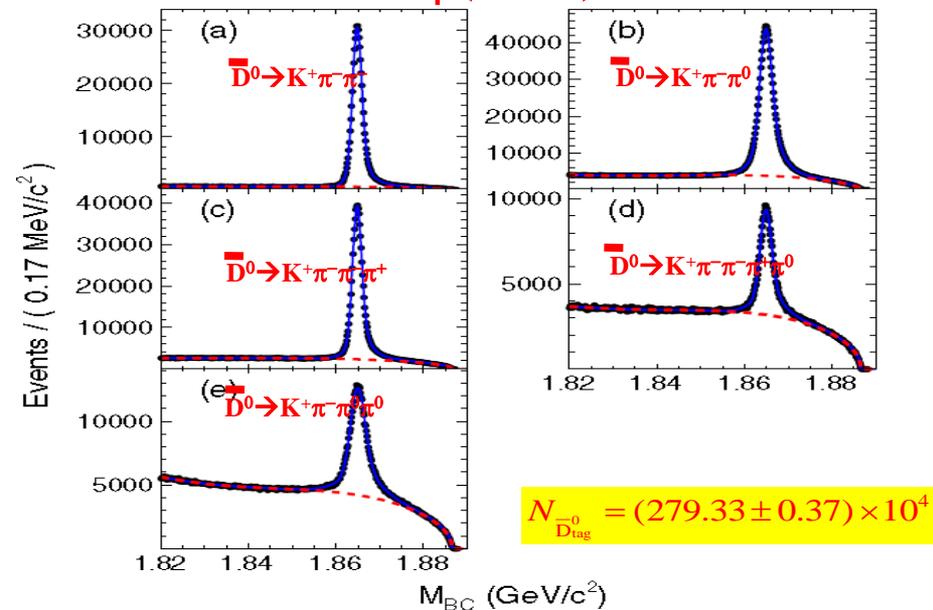
$$|V_{cd}| = 0.2210 \pm 0.0058 \pm 0.0047$$

# Comparisons of $B[D^+ \rightarrow \mu^+ \nu_\mu]$ and $f_{D^+}$

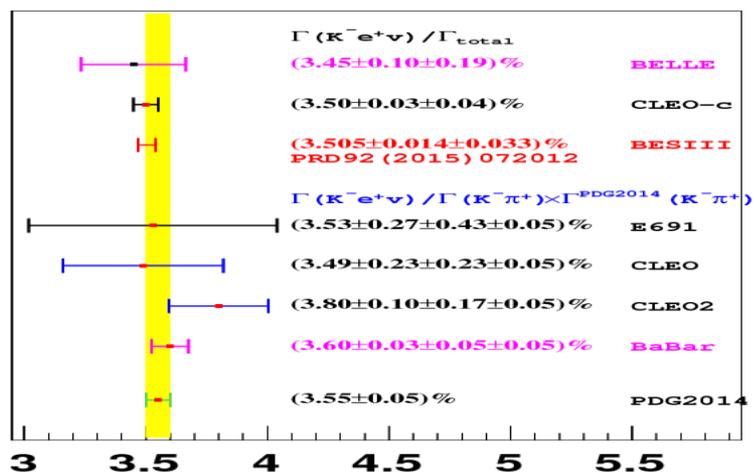
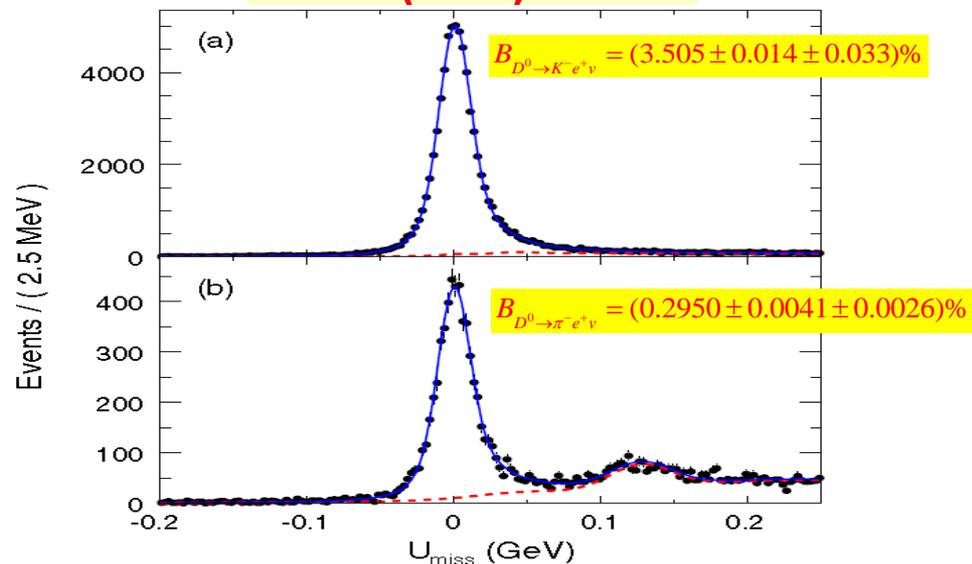


# Measurement of $B[D^0 \rightarrow K(\pi)^- e^+ \nu]$

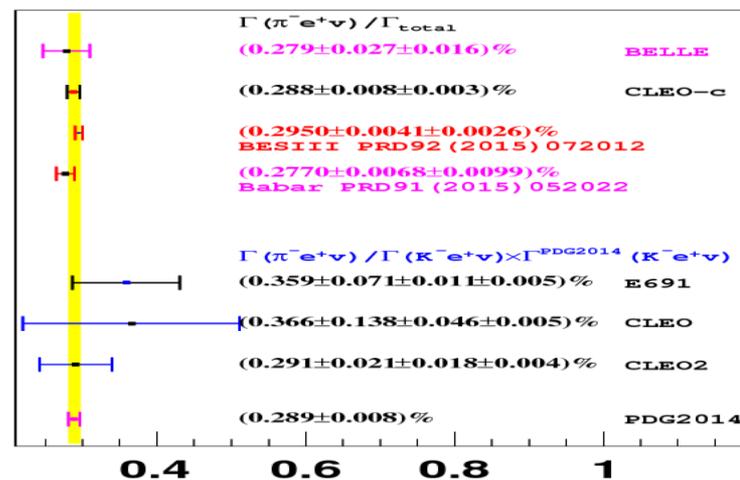
$e^+e^- \rightarrow \psi(3770) \rightarrow D^0 \bar{D}^0$



PRD92(2015)072012

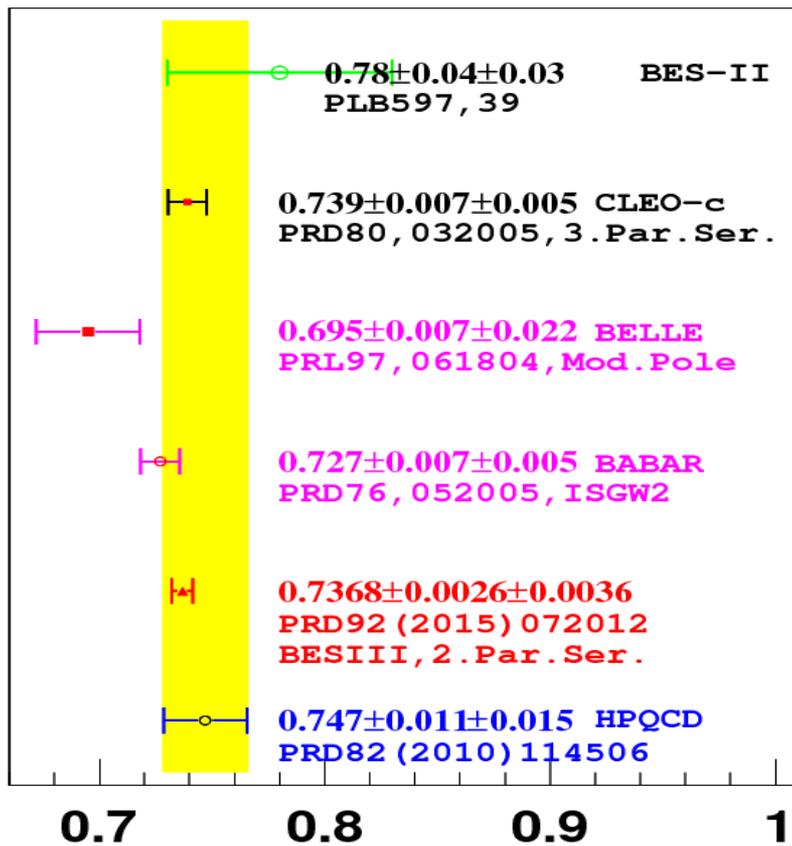


$B[D^0 \rightarrow K^- e^+ \nu]$

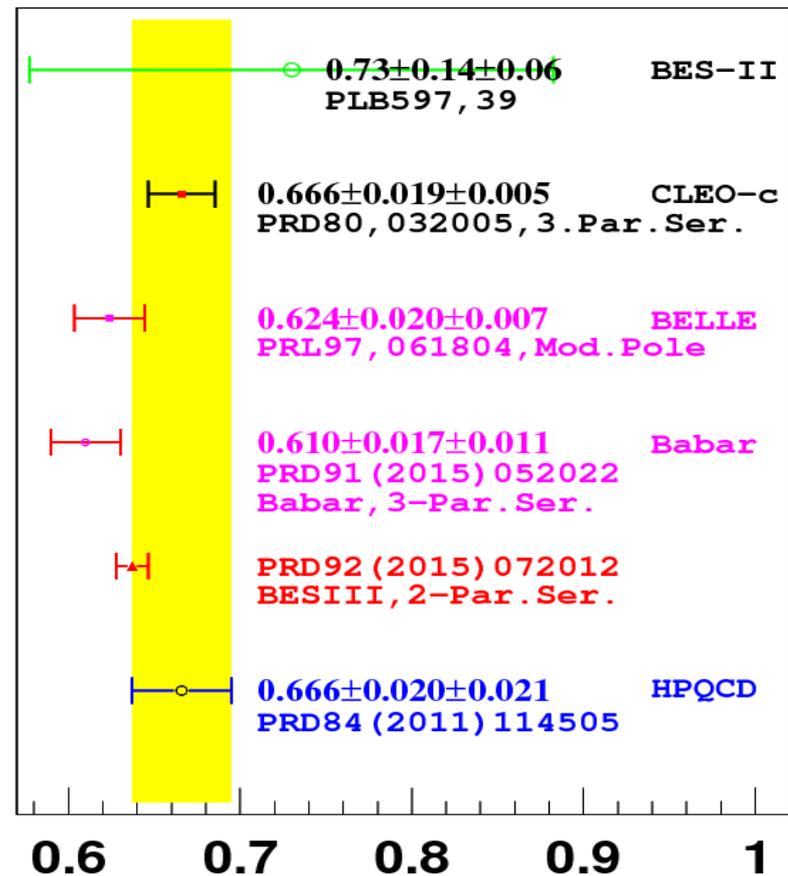


$B[D^0 \rightarrow \pi^- e^+ \nu]$

# Measurement of $f_+^{K(\pi)}(0)$



$f_+^{K(0)}$



$f_+^{\pi(0)}$

# Analysis of $D^+ \rightarrow K_L e^+ \nu$

[PRD92(2015) 112008]

➤ Regardless of long flight distance,  $K_L$  interact with EMC and deposit part of energy, thus giving position information

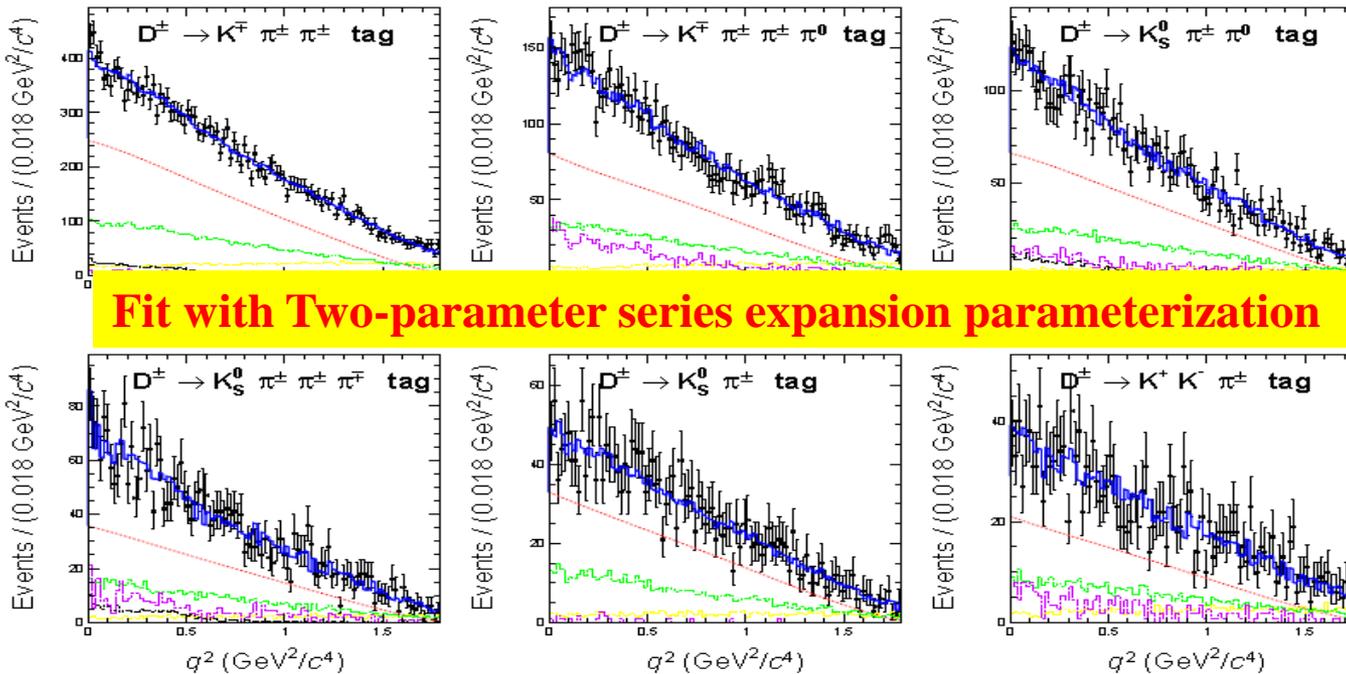
➤ After reconstructing all other particles,  $K_L$  can be inferred with position information and constraint  $U_{\text{miss}} \rightarrow 0$

$$\mathcal{B}(D^+ \rightarrow K_L e^+ \nu) = (4.482 \pm 0.027 \pm 0.103)\%$$

$$A_{CP} \equiv \frac{\mathcal{B}(D^+ \rightarrow K_L^0 e^+ \nu_e) - \mathcal{B}(D^- \rightarrow K_L^0 e^- \bar{\nu}_e)}{\mathcal{B}(D^+ \rightarrow K_L^0 e^+ \nu_e) + \mathcal{B}(D^- \rightarrow K_L^0 e^- \bar{\nu}_e)}$$

$$A_{CP}^{D^+ \rightarrow K_L e^+ \nu} = (-0.59 \pm 0.60 \pm 1.50)\%$$

## Simultaneous fit to observed numbers of DT candidates



The first measurement of the BR for  $D^+ \rightarrow K_L e^+ \nu_e$

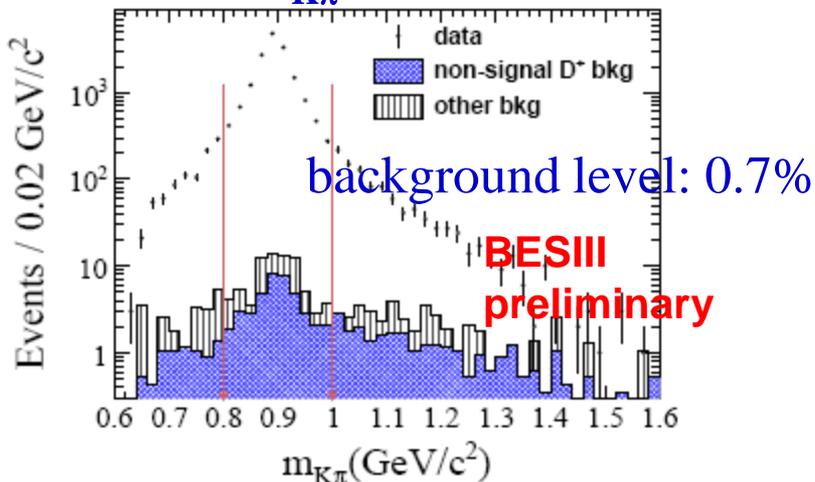
Fit with Two-parameter series expansion parameterization

$$f_{K^+}^K(0) |V_{cs}| = 0.728 \pm 0.006 \pm 0.011$$

# Analysis of $D^+ \rightarrow K^- \pi^+ e^+ \nu$

arXiv:1512.08627

## $M_{K\pi}$ distribution



$$B(D^+ \rightarrow K^- \pi^+ e^+ \nu_e) = (3.71 \pm 0.03 \pm 0.08)\%$$

$$B(D^+ \rightarrow K^- \pi^+ e^+ \nu_e)_{[0.8,1]} = (3.33 \pm 0.03 \pm 0.07)\%$$

## Fit Results (Preliminary)

### ■ Fitted fractions of the component

$$f(D^+ \rightarrow (K^- \pi^+)_{K^{*0}(892)} e^+ \nu_e) = (93.93 \pm 0.22 \pm 0.18)\%$$

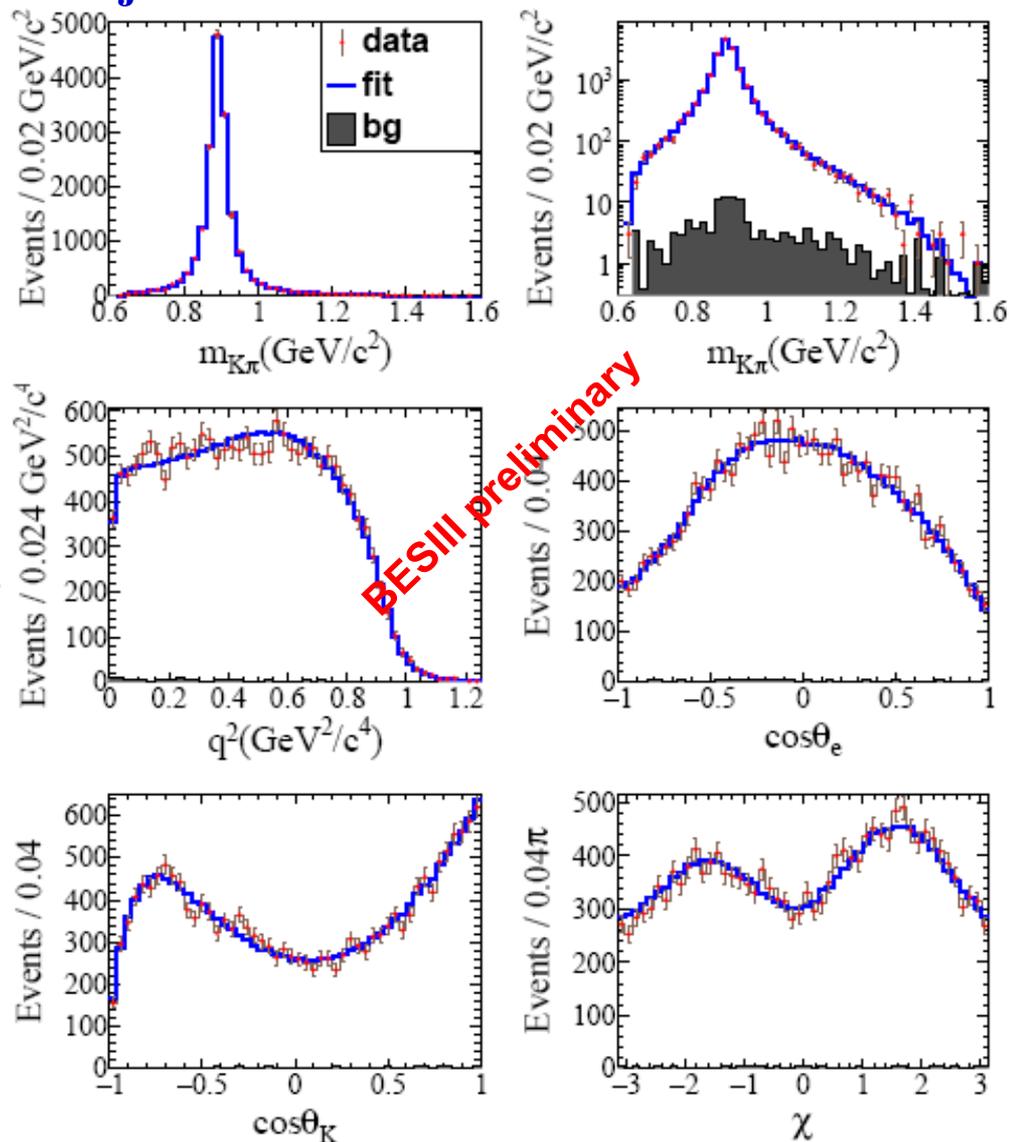
$$f(D^+ \rightarrow (K^- \pi^+)_{S\text{-wave}} e^+ \nu_e) = (6.05 \pm 0.22 \pm 0.18)\%$$

### ■ Parameters of $K^{*0}(892)$

$$m_{K^{*0}(892)} = (894.60 \pm 0.25 \pm 0.08) \text{ MeV}/c^2$$

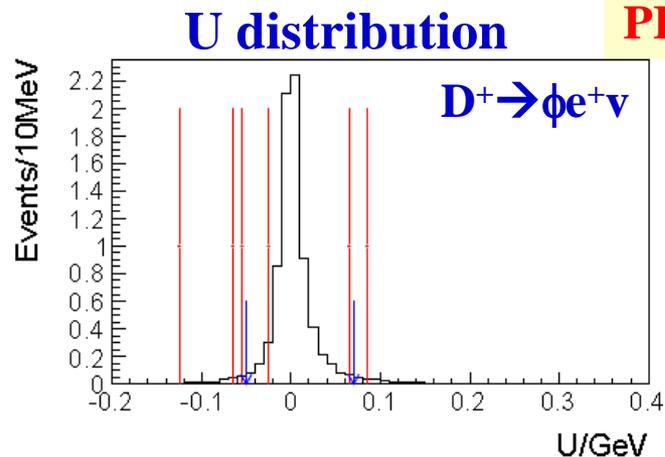
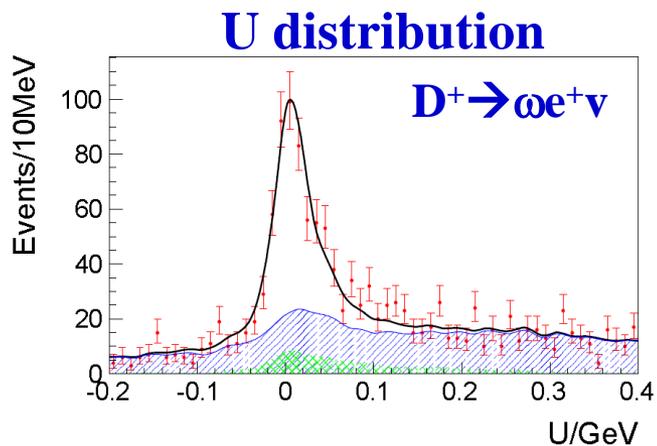
$$\Gamma_{K^{*0}(892)} = (46.42 \pm 0.56 \pm 0.15) \text{ MeV}/c^2$$

## Projections of data and fitted MC distributions



# Study of $D^+ \rightarrow \omega e^+ \nu$ and search for $D^+ \rightarrow \phi e^+ \nu$

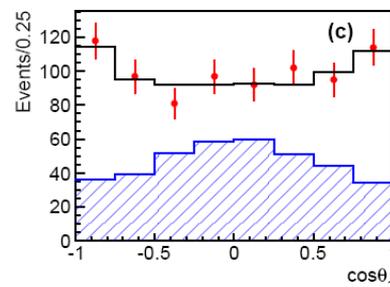
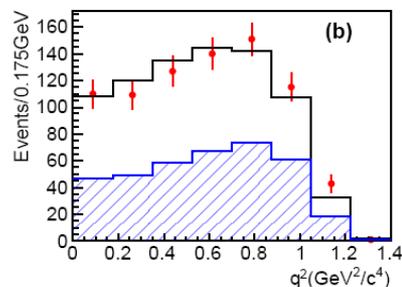
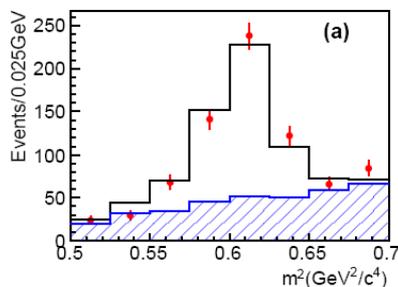
PRD92(2015) 071101R



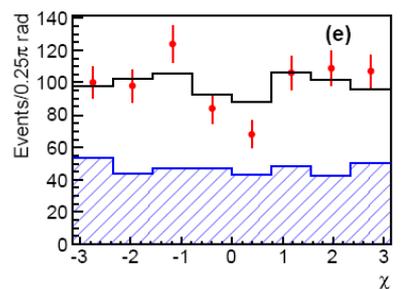
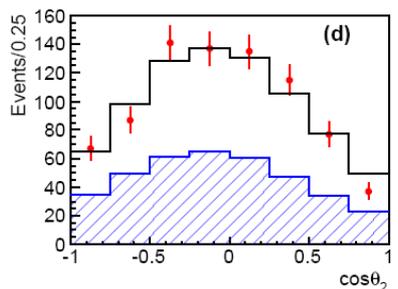
Red dots: data;  
Arrows: signal region.

| Mode               | This work                                 | Previous                                  |
|--------------------|---|---|
| $\omega e^+ \nu_e$ | $(1.63 \pm 0.11 \pm 0.08) \times 10^{-3}$ | $(1.82 \pm 0.18 \pm 0.07) \times 10^{-3}$ |
| $\phi e^+ \nu_e$   | $< 1.3 \times 10^{-5}$ (90% C.L.)         | $< 9.0 \times 10^{-5}$ (90% C.L.)         |

**Better precision  
or sensitivity**



Amplitude analysis of  $D^+ \rightarrow \omega e^+ \nu$  is performed for the first time



**Results of form factor ratios:**

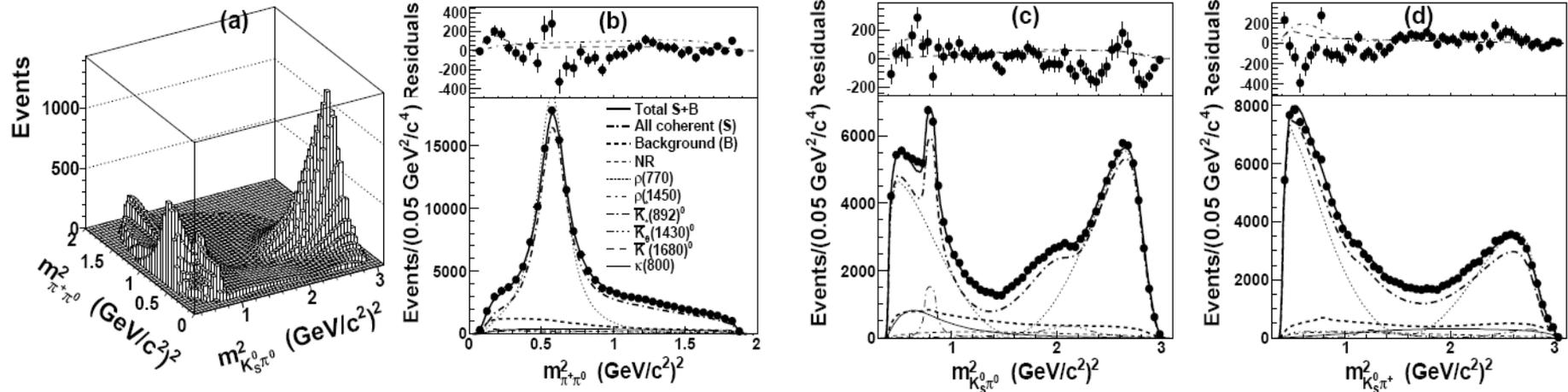
$$r_V = V(0)/A_1(0) = 1.24 \pm 0.09 \pm 0.06$$

$$r_2 = A_2(0)/A_1(0) = 1.06 \pm 0.15 \pm 0.05$$

# Dalitz Plot Analysis of $D^+ \rightarrow K_S^0 \pi^+ \pi^0$

Distribution of (a) fitted p.d.f, and projections on (b)  $M^2_{\pi^+\pi^0}$ , (c)  $M^2_{K_S^0\pi^0}$ , and (d)  $M^2_{K_S^0\pi^+}$

PRD89(2014) 052001



Partial BFs calculated by combining fitted fractions with PDG's  $B[D^+ \rightarrow K_S^0 \pi^+ \pi^0]$ .

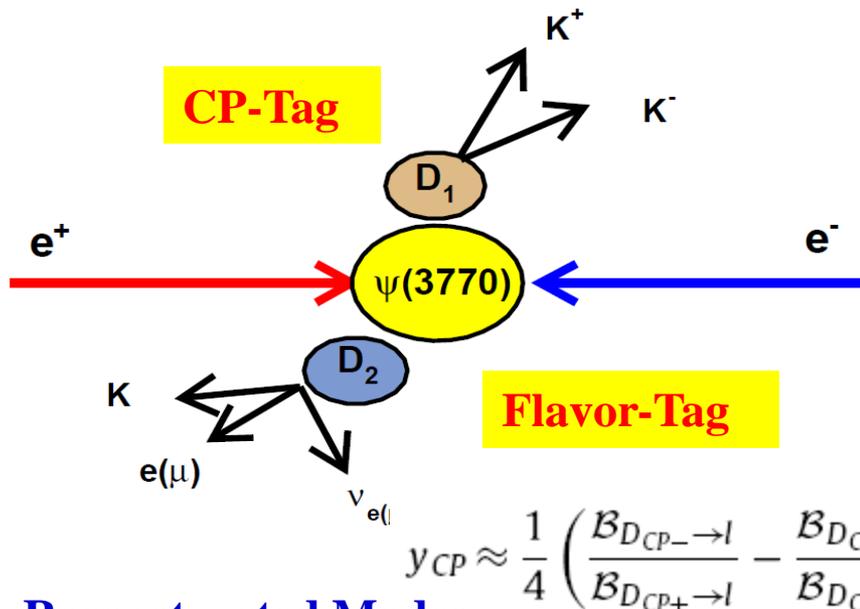
| Mode   | Partial Branching Fraction (%)                |
|--|---|
| $D^+ \rightarrow K_S^0 \pi^+ \pi^0$ Non Resonant   | $0.32 \pm 0.05 \pm 0.25^{+0.28}_{-0.25}$      |
| $D^+ \rightarrow \rho^+ K_S^0, \rho^+ \rightarrow \pi^+ \pi^0$                                     | $5.83 \pm 0.16 \pm 0.30^{+0.45}_{-0.15}$      |
| $D^+ \rightarrow \rho(1450)^+ K_S^0, \rho(1450)^+ \rightarrow \pi^+ \pi^0$                         | $0.15 \pm 0.02 \pm 0.09^{+0.07}_{-0.11}$      |
| $D^+ \rightarrow \overline{K}^*(892)^0 \pi^+, \overline{K}^*(892)^0 \rightarrow K_S^0 \pi^0$       | $0.250 \pm 0.012 \pm 0.015^{+0.025}_{-0.024}$ |
| $D^+ \rightarrow \overline{K}_0^*(1430)^0 \pi^+, \overline{K}_0^*(1430)^0 \rightarrow K_S^0 \pi^0$ | $0.26 \pm 0.04 \pm 0.05 \pm 0.06$             |
| $D^+ \rightarrow \overline{K}^*(1680)^0 \pi^+, \overline{K}^*(1680)^0 \rightarrow K_S^0 \pi^0$     | $0.09 \pm 0.01 \pm 0.05^{+0.04}_{-0.08}$      |
| $D^+ \rightarrow \overline{K}^0 \pi^+, \overline{K}^0 \rightarrow K_S^0 \pi^0$                     | $0.54 \pm 0.09 \pm 0.28^{+0.36}_{-0.19}$      |
| $NR + \overline{K}^0 \pi^+$  | $1.30 \pm 0.12 \pm 0.12^{+0.12}_{-0.30}$      |
| $K_S^0 \pi^0$ S-wave   | $1.21 \pm 0.10 \pm 0.16^{+0.19}_{-0.27}$      |

**Partial branching ratios are measured with higher precision than previous measurements.**

# D $\bar{D}$ mixing parameter $y_{CP}$

PLB 744, 339 (2015)

We measure the  $y_{CP}$  using CP-tagged semi-leptonic D decays, which allows to access CP asymmetry in mixing and decays.

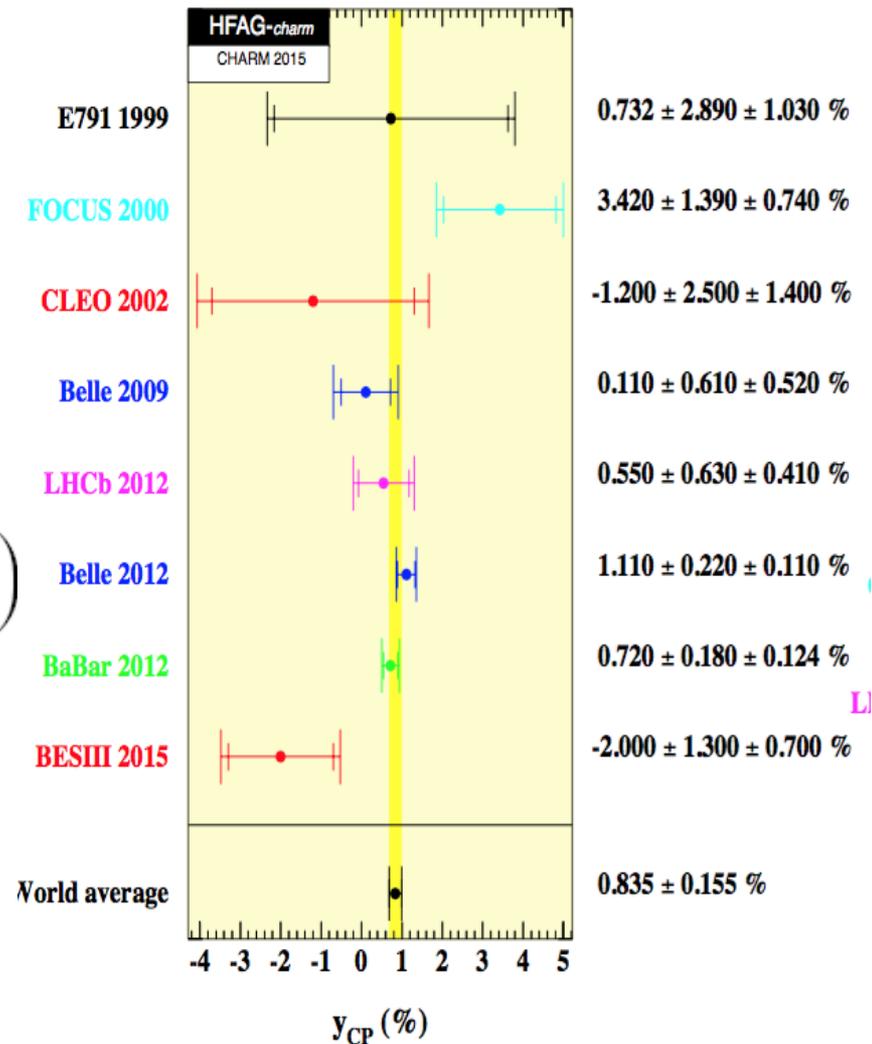


## Reconstructed Modes:

| Type         | Mode                                  |
|--------------|---------------------------------------|
| CP+          | $K^+K^-, \pi^+\pi^-, K_S^0\pi^0\pi^0$ |
| CP-          | $K_S^0\pi^0, K_S^0\omega, K_S^0\eta$  |
| Semileptonic | $K^\mp e^\pm\nu, K^\mp \mu^\pm\nu$    |

$$y_{CP} = (-2.1 \pm 1.3 \pm 0.7)\%$$

Compatible with the previous measurements.

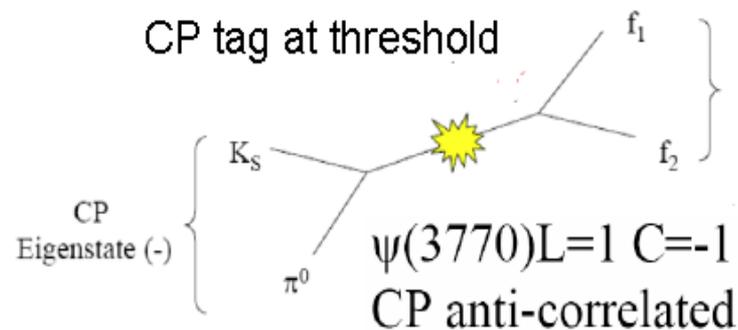


# Strong phase difference $\delta_{K\pi}$

PLB 734, 227 (2014)

Quantum correlation  $\rightarrow$  Interference  $\rightarrow$  access strong phase!

If CP violation in charm is neglected: mass eigenstates = CP eigenstates

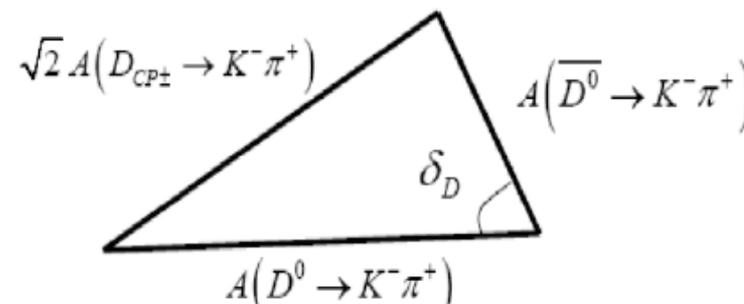


$$\mathcal{A}_{K\pi}^{\text{CP}} \equiv \frac{\mathcal{B}_{D^{S-} \rightarrow K^- \pi^+} - \mathcal{B}_{D^{S+} \rightarrow K^- \pi^+}}{\mathcal{B}_{D^{S-} \rightarrow K^- \pi^+} + \mathcal{B}_{D^{S+} \rightarrow K^- \pi^+}}$$

$$2r \cos \delta_{K\pi} + y = (1 + R_{\text{WS}}) \cdot \mathcal{A}_{\text{CP} \rightarrow K\pi}$$

$$|D_1\rangle \equiv \frac{|D^0\rangle + |\bar{D}^0\rangle}{\sqrt{2}} \quad |D_2\rangle \equiv \frac{|D^0\rangle - |\bar{D}^0\rangle}{\sqrt{2}}$$

$$\mathcal{A}_{\text{CP}}^{K\pi} = (12.7 \pm 1.3 \pm 0.7) \times 10^{-2}$$



$\delta_{K\pi}$  is important to relate to mixing parameters  $x$  and  $y$  from  $x'$  and  $y'$

Reconstructed Modes:

| Type     | Mode   |
|----------|--|
| Flavored | $K^- \pi^+, K^+ \pi^-$   |
| S+       | $K^+ K^-, \pi^+ \pi^-, K_S^0 \pi^0 \pi^0, \pi^0 \pi^0, \rho^0 \pi^0$ |
| S-       | $K_S^0 \pi^0, K_S^0 \eta, K_S^0 \omega$                              |

With external inputs of the parameters in HFAG2013 and PDG

$$R_D = 3.47 \pm 0.06\%, \quad y = 6.6 \pm 0.9\% \quad R_{\text{WS}} = 3.80 \pm 0.05\%$$

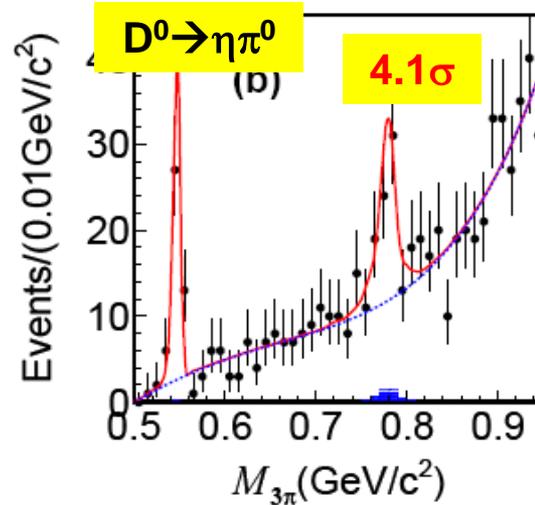
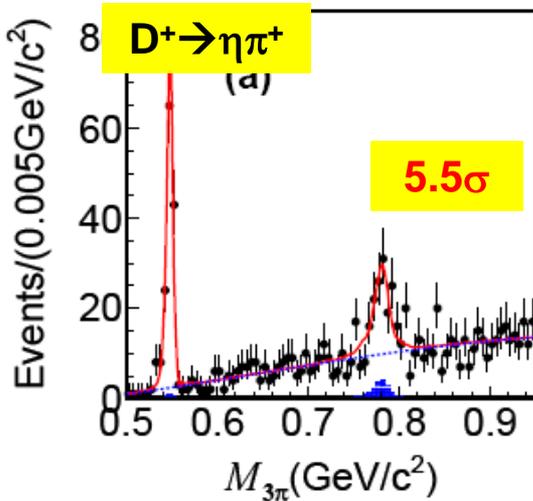
$$\cos \delta_{K\pi} = 1.02 \pm 0.11 \pm 0.06 \pm 0.01$$

most precise to date

# Observation/Evidence for SCS decay $D^{+(0)} \rightarrow \omega \pi^{+(0)}$

arXiv:1512.06998

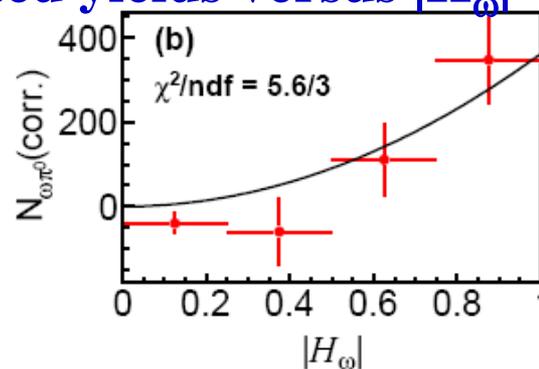
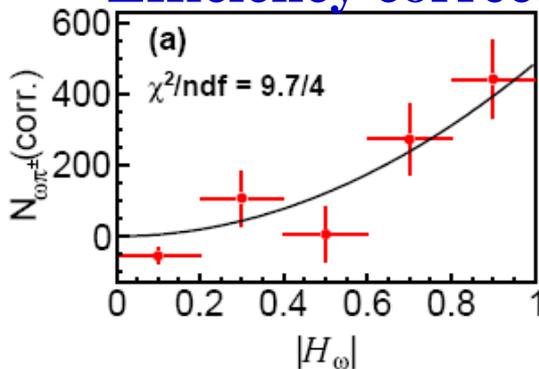
## Suppress background via DT method



➤ Predications of  $B[D \rightarrow \omega \pi]$  at  $1.0 \times 10^{-4}$  level.

➤  $D \rightarrow \omega \pi$  were studied at CLEO-c with ST method, but only set BF upper limits

## Efficiency corrected yields versus $|H_\omega|$



## Summary of BF's measurements

| Mode                           | This work                                 | Previous measurements              |
|--------------------------------|---|------------------------------------|
| $D^+ \rightarrow \omega \pi^+$ | $(2.79 \pm 0.57 \pm 0.16) \times 10^{-4}$ | $< 3.4 \times 10^{-4}$ at 90% C.L. |
| $D^0 \rightarrow \omega \pi^0$ | $(1.17 \pm 0.34 \pm 0.07) \times 10^{-4}$ | $< 2.6 \times 10^{-4}$ at 90% C.L. |
| $D^+ \rightarrow \eta \pi^+$   | $(3.07 \pm 0.22 \pm 0.13) \times 10^{-3}$ | $(3.53 \pm 0.21) \times 10^{-3}$   |
| $D^0 \rightarrow \eta \pi^0$   | $(0.65 \pm 0.09 \pm 0.04) \times 10^{-3}$ | $(0.68 \pm 0.07) \times 10^{-3}$   |

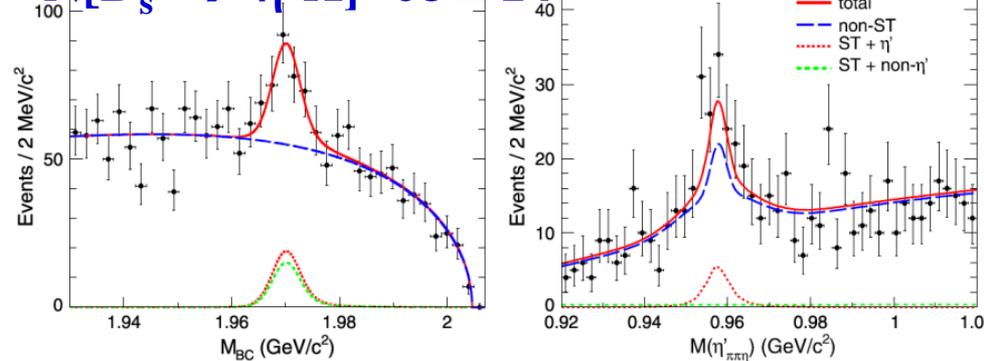
**Improve understanding of U-spin and SU(3) flavor symmetry breaking effects in D decays and benefitting theoretical prediction of CP violation in D decays**

# $D_s^+ \rightarrow \eta' X$ and $\eta' \rho^+$

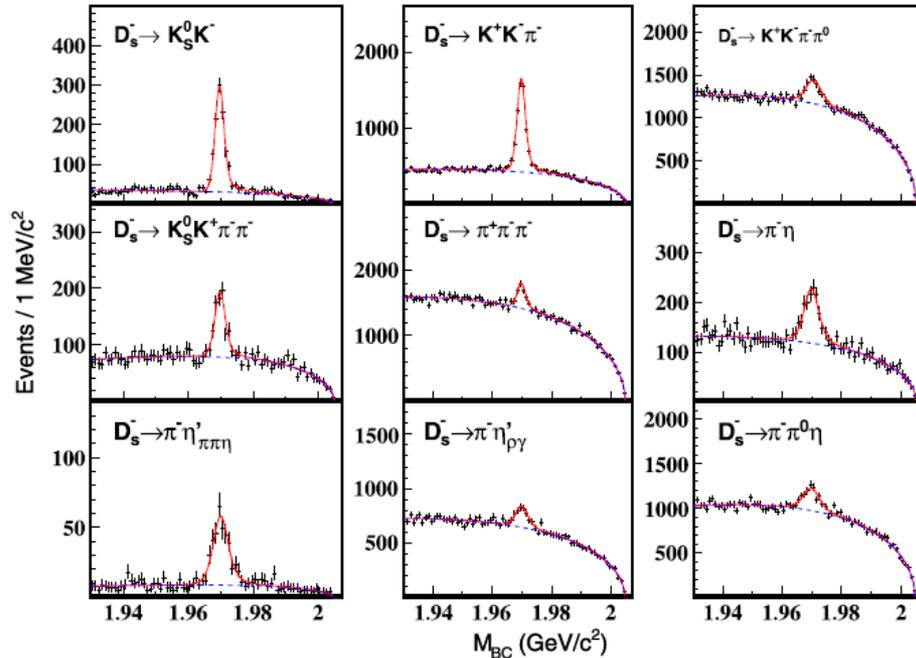
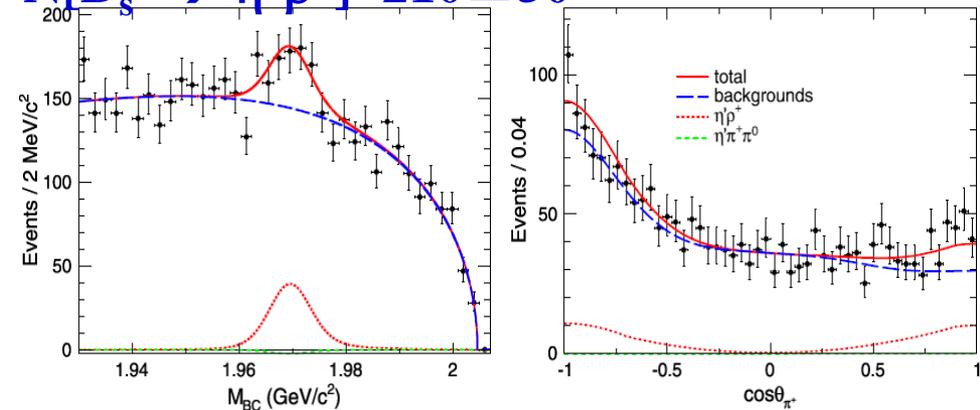
About 15.6 K ST  $D_s^-$  events by using 9 ST modes

PLB750 466(2015)

$N[D_s^+ \rightarrow \eta' X] = 68 \pm 14$



$N[D_s^+ \rightarrow \eta' \rho^+] = 210 \pm 50$



$B[D_s^+ \rightarrow \eta' X] = (8.8 \pm 1.8 \pm 0.5)\%$

Consistent with CLEO measurements  $B[D_s^+ \rightarrow \eta' X] = (11.7 \pm 1.8)\%$  [PRD79 112008(2009)]

$B[D_s^+ \rightarrow \eta' \rho^+] = (5.8 \pm 1.4 \pm 0.4)\%$   $B^{\text{exp}}[D_s^+ \rightarrow \eta' \rho^+] = (3.0 \pm 0.5)\%$  [PRD84 074019(2011)]

Resolve the disagreement between theoretical predication and CLEO-c's previous measurement.  $B[D_s^+ \rightarrow \eta' \rho^+] = (12.5 \pm 2.2)\%$  [PRD58 052002(1998)]

# Absolute BF for $\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e$

Theoretical calculations on the BF ranges from 1.4% to 9.2%

PDG2014:  $(2.1 \pm 0.6)\%$

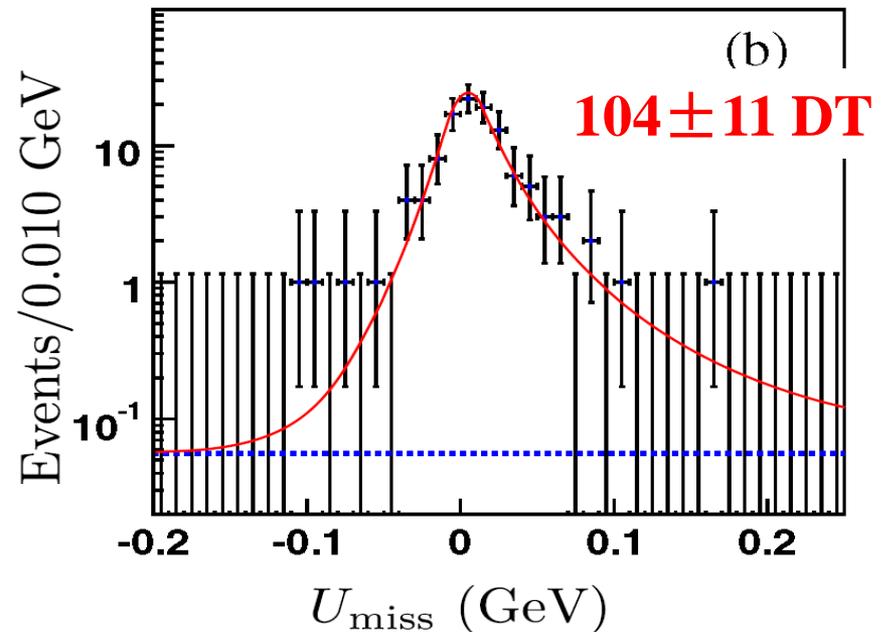
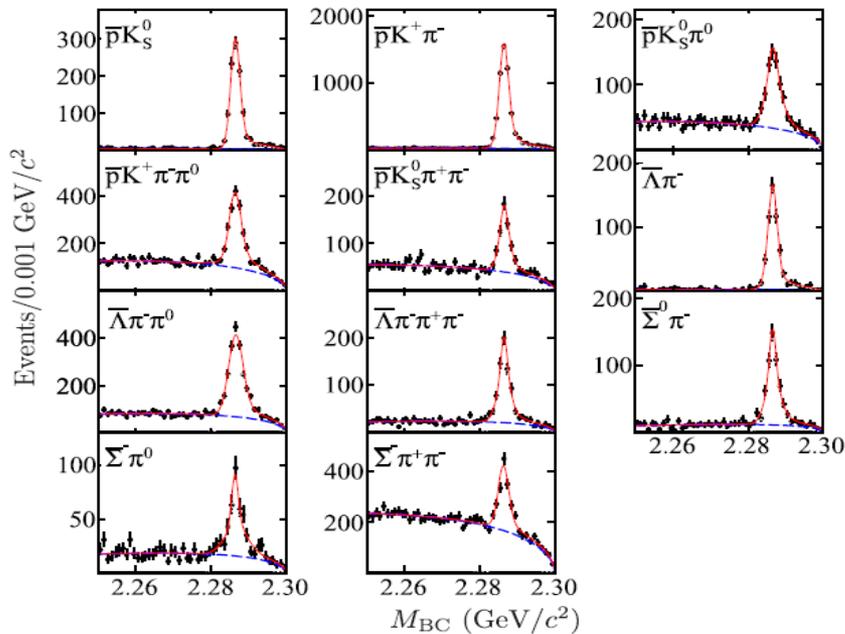
PDG2015:  $(2.9 \pm 0.5)\%$



Input  $B[\Lambda_c^+ \rightarrow pK^- \pi^+] = (6.84^{+0.32}_{-0.40})\%$   
by BELLE [PRL113,042002(2014)]

$14415 \pm 159$  events with 11 ST modes

PRL115(2015)221805



$B[\Lambda_c^+ \rightarrow \Lambda e^+ \nu] = (3.63 \pm 0.38 \pm 0.20)\%$  First absolute measurement

Important for test and calibrate the LQCD calculations.

# Absolute BFs for $\Lambda_c^+$ hadron decays

Measurement using the threshold pair-productions via  $e^+e^-$  annihilation is unique: the most simple and straightforward

arXiv:1511.08380

Accepted by PRL

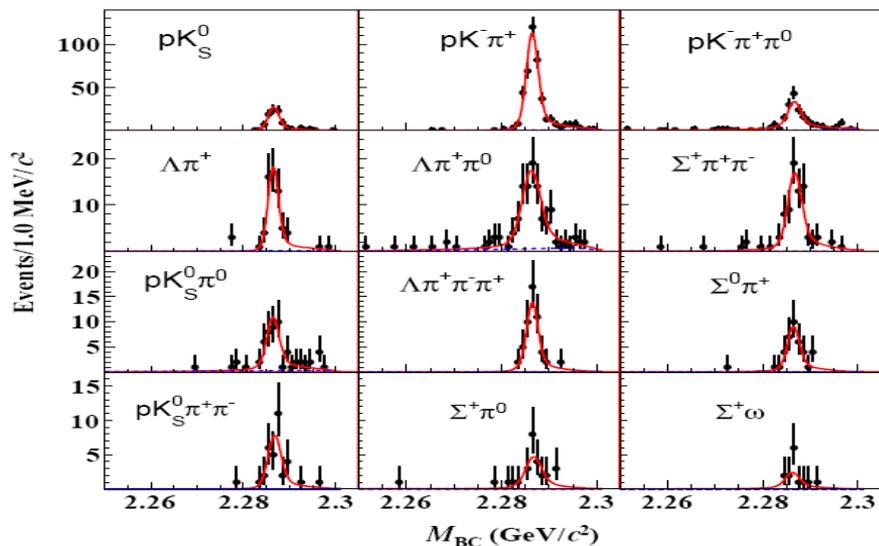
A global least-square fitter is utilized to improve the measured precision for 12  $\Lambda_c^+$  hadronic decay channels.

$$N_{-j}^{DT} = \sum_{i^+ \neq i} N_{i^+j^-}^{DT} + \sum_{i^- \neq i} N_{i^-j^+}^{DT} + N_{jj}^{DT}$$

✓ Absolute BFs are improved significantly.

✓ BESIII BF for  $\Lambda_c^+ \rightarrow pK^- \pi^+$  is smaller.

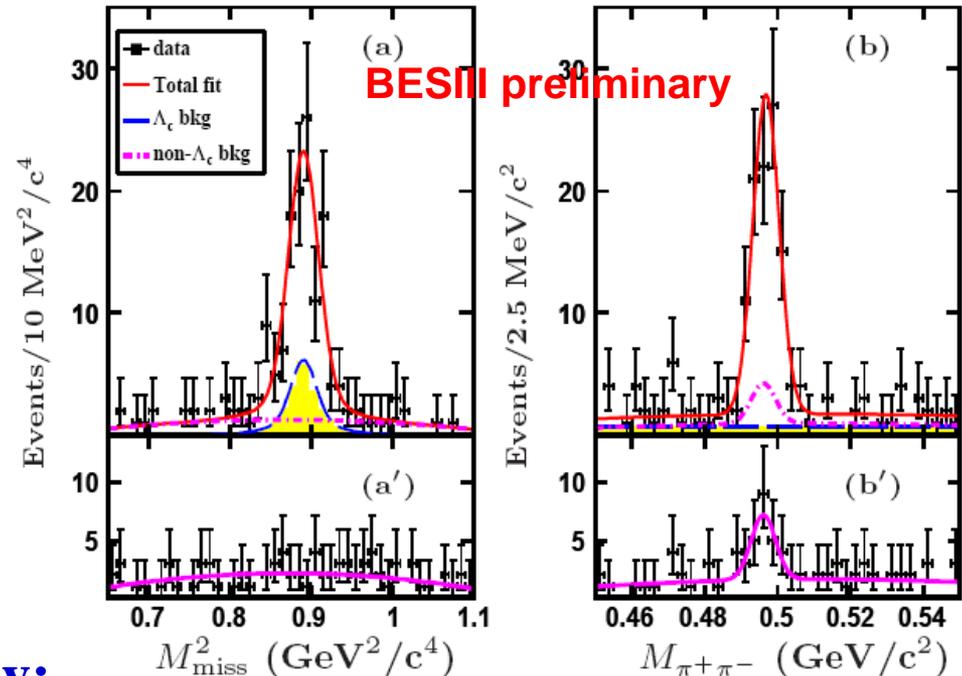
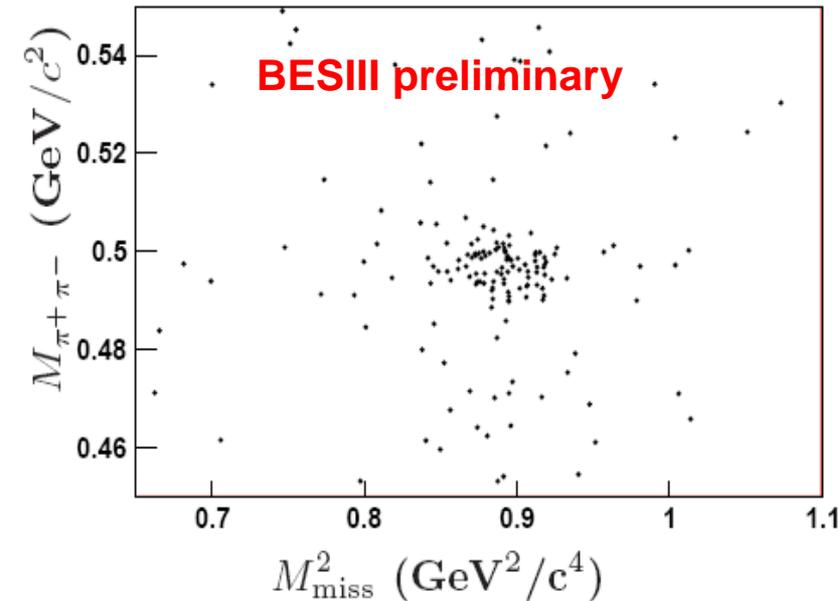
✓ Improved absolute BF of  $pK^- \pi^+$  together with BELLE's result are key to calibrate other decays.



| Mode                        | This work (%)            | PDG (%)         | Belle $\mathcal{B}$             |
|-----------------------------|--------------------------|-----------------|---------------------------------|
| $pK_S^0$                    | $1.52 \pm 0.08 \pm 0.03$ | $1.15 \pm 0.30$ |                                 |
| $pK^- \pi^+$                | $5.84 \pm 0.27 \pm 0.23$ | $5.0 \pm 1.3$   | $6.84 \pm 0.24^{+0.21}_{-0.27}$ |
| $pK_S^0 \pi^0$              | $1.87 \pm 0.13 \pm 0.05$ | $1.65 \pm 0.50$ |                                 |
| $pK_S^0 \pi^+ \pi^-$        | $1.53 \pm 0.11 \pm 0.09$ | $1.30 \pm 0.35$ |                                 |
| $pK^- \pi^+ \pi^0$          | $4.53 \pm 0.23 \pm 0.30$ | $3.4 \pm 1.0$   |                                 |
| $\Lambda \pi^+$             | $1.24 \pm 0.07 \pm 0.03$ | $1.07 \pm 0.28$ |                                 |
| $\Lambda \pi^+ \pi^0$       | $7.01 \pm 0.37 \pm 0.19$ | $3.6 \pm 1.3$   |                                 |
| $\Lambda \pi^+ \pi^- \pi^+$ | $3.81 \pm 0.24 \pm 0.18$ | $2.6 \pm 0.7$   |                                 |
| $\Sigma^0 \pi^+$            | $1.27 \pm 0.08 \pm 0.03$ | $1.05 \pm 0.28$ |                                 |
| $\Sigma^+ \pi^0$            | $1.18 \pm 0.10 \pm 0.03$ | $1.00 \pm 0.34$ |                                 |
| $\Sigma^+ \pi^+ \pi^-$      | $4.25 \pm 0.24 \pm 0.20$ | $3.6 \pm 1.0$   |                                 |
| $\Sigma^+ \omega$           | $1.56 \pm 0.20 \pm 0.07$ | $2.7 \pm 1.0$   |                                 |

# Observation of $\Lambda_c^+ \rightarrow n K_S^0 \pi^+$

First observation of  $\Lambda_c^+$  decays to final states involving the neutron.



The missing neutron is detected by:

$$M_{\text{miss}}^2 = (p_{\Lambda_c^+} - p_{K_S^0} - p_{\pi^+})^2 = E_{\text{miss}}^2 - c^2 |\vec{p}_{\text{miss}}|^2$$

**$83 \pm 11$  net signal events**

**BESIII Preliminary results:**

$$B[\Lambda_c^+ \rightarrow n K_S^0 \pi^+] = (1.82 \pm 0.23 \pm 0.11)\%$$

Fit to  $M_{\text{miss}}^2$  and  $M_{\pi^+\pi^-}$  spectra in (a,b)  $\Lambda_c^-$  signal region and (a',b')  $\Lambda_c^-$  sideband region simultaneously.

The relative BF of neutron-involved mode to proton-involved mode is essential to test the isospin symmetry for  $\Lambda_c^+$  decays.  
[arXiv:1601.04241]

# Summary

□ **BESIII provides important results on charm decays**

➤ **leptonic and semi-leptonic decays**

➤ **D hadronic decays**

➤  **$D^0$  mixing and strong phase**

➤  **$\Lambda_c^+$  hadronic and semi-leptonic decays**

**important to test LQCD calculations, CKM matrix UT, search for NP beyond SM**

□ **More fruitful results will come out!**

**Thanks!**

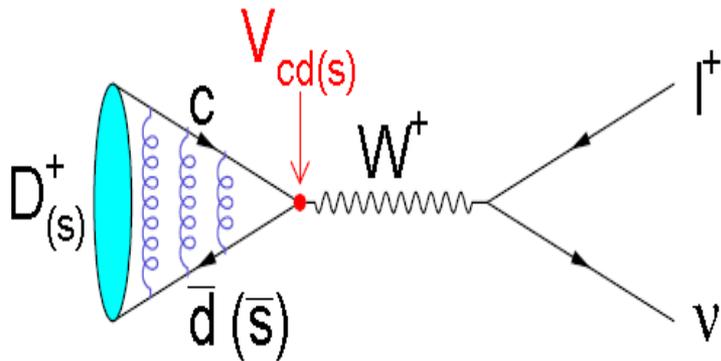
# Introduction

**Precision measurement of charm decays provide rich information to probe for strong and weak effects**

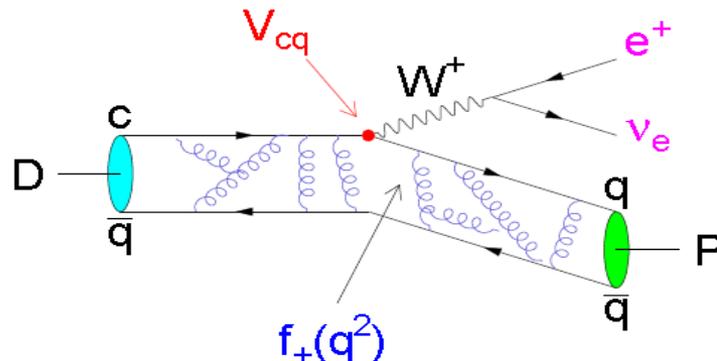
- **Unitarity test of CKM matrix: direct access quark mixing matrix element  $|V_{cs(d)}|$  or strong phase constrained  $\gamma/\phi_3$**
- **LQCD calibration: precise decay constant  $f_{D(s)+}$ , form factors  $f_{D \rightarrow K(\pi)}(q^2)$  and others**
- **New physics BSM: evidence of rare decay/CP violation, or significant deviation of CKM unitarity/LQCD calculation**
- **Important inputs for beauty physics: Significantly improved decay rates or dynamics**

# D leptonic and semi-leptonic decays

Bridge to extract  $D_{(s)}^+$  decay constant(s)  $f_{D_{(s)}^+}$ , form factors  $f_+^{D \rightarrow K(\pi)}(q^2)$  and quark mixing matrix elements  $|V_{cs(d)}|$



$$\Gamma(D_{(s)}^+ \rightarrow l^+ \nu_l) = \frac{G_F^2 f_{D_{(s)}^+}^2}{8\pi} |V_{cd(s)}|^2 m_l^2 m_{D_{(s)}^+} \left(1 - \frac{m_l^2}{m_{D_{(s)}^+}^2}\right)^2$$



$$\frac{d\Gamma}{dq^2} = X \frac{G_F^2 |V_{cd(s)}|^2}{24\pi^3} p^3 |f_+(q^2)|^2$$

■ Improved  $f_{D_{(s)}^+}$ ,  $f_+^{D \rightarrow K(\pi)}(q^2)$  of D semi-leptonic decays calibrate LQCD calculations at higher accuracy. Once they pass experimental test, the precise LQCD calculations of  $f_D/f_B$ ,  $f_{D_s}/f_{B_s}$  and form factor ratios are helpful for measurements in B decays

■ Recent LQCD calculations on  $f_{D_{(s)}^+}$  [0.5(0.5)%],  $f_+^{D \rightarrow K(\pi)}(0)$  [1.7(4.4)%] provide good chance to precisely measure  $|V_{cs(d)}|$

$$\begin{aligned}
\frac{d\Gamma}{dq^2 d\cos\theta_1 d\cos\theta_2 d\chi dm_{\pi\pi\pi}} &= \frac{3}{8(4\pi)^4} G_F^2 |V_{cd}|^2 \frac{p_\omega q^2}{M_D^2} \mathcal{B}(\omega \rightarrow \pi\pi\pi) |\mathcal{BW}(m_{\pi\pi\pi})|^2 \\
&[(1 + \cos\theta_2)^2 \sin^2\theta_1 |H_+(q^2, m_{\pi\pi\pi})|^2 \\
&+ (1 - \cos\theta_2)^2 \sin^2\theta_1 |H_-(q^2, m_{\pi\pi\pi})|^2 + 4 \sin^2\theta_2 \cos^2\theta_1 |H_0(q^2, m_{\pi\pi\pi})|^2 \\
&+ 4 \sin\theta_2 (1 + \cos\theta_2) \sin\theta_1 \cos\theta_1 \cos\chi H_+(q^2, m_{\pi\pi\pi}) H_0(q^2, m_{\pi\pi\pi}) \\
&- 4 \sin\theta_2 (1 - \cos\theta_2) \sin\theta_1 \cos\theta_1 \cos\chi H_-(q^2, m_{\pi\pi\pi}) H_0(q^2, m_{\pi\pi\pi}) \\
&- 2 \sin^2\theta_2 \sin^2\theta_1 \cos 2\chi H_+(q^2, m_{\pi\pi\pi}) H_-(q^2, m_{\pi\pi\pi})].
\end{aligned}$$

$$\begin{aligned}
H_\pm(q^2) &= M A_1(q^2) \mp 2 \frac{M_{DP\omega}}{M} V(q^2) \\
H_0(q^2) &= \frac{1}{2m_{\pi\pi\pi} \sqrt{q^2}} [(M_D^2 - m_{\pi\pi\pi}^2 - q^2) M A_1(q^2) \\
&- 4 \frac{M_{DP\omega}^2}{M} A_2(q^2)] \quad (3)
\end{aligned}$$

where  $M = M_D + m_{\pi\pi\pi}$ . For the  $q^2$  dependence, a single

pole parameterization [24] is applied:

$$V(q^2) = \frac{V(0)}{1 - q^2/m_V^2}, \quad A_{1,2}(q^2) = \frac{A_{1,2}(0)}{1 - q^2/m_A^2}, \quad (4)$$

where the pole masses  $m_V$  and  $m_A$  are expected to be close to  $M_{D^*(1-)} = 2.01 \text{ GeV}/c^2$  and  $M_{D^*(1+)} = 2.42 \text{ GeV}/c^2$  [14] for the vector and axial form factors, respectively. The ratios of these form factors, evaluated at  $q^2 = 0$ ,  $r_V = \frac{V(0)}{A_1(0)}$  and  $r_2 = \frac{A_2(0)}{A_1(0)}$ , are measured in this paper.