Physics with entangled neutral $D$ mesons at $\text{BES}^{\text{III}}$ and future perspectives

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Outline

• Introduction
• The BESIII experiment
• Physics with entangled neutral D mesons
• Recent BESIII charm results
• Summary & prospectives
Neutral $D$ meson oscillation

- $D_0$ and $\bar{D}_0$ can transform into each other (like Kaons an Bs)
- The mass eigenstates are
  \[
  |D_1\rangle = p |D^0\rangle + q |\bar{D}^0\rangle \\
  |D_2\rangle = p |D^0\rangle - q |\bar{D}^0\rangle
  \]
- With eigenvalues
  \[
  \mu_1 = m_1 - \frac{i}{2} \Gamma_1 \\
  \mu_2 = m_2 - \frac{i}{2} \Gamma_2
  \]

Short-distance:
- CKM suppression: $b$
- GIM suppression: $d, s$

Long-distance:
- CKM suppression: $b$
- GIM suppression: $d, s$

$\chi$ mixing: Channel for New Physics.

$y$ (long-distance) mixing: SM background.

$$
\begin{align*}
\mu_1 &= m_1 - \frac{i}{2} \Gamma_1 \\
\mu_2 &= m_2 - \frac{i}{2} \Gamma_2 \\
m &= \frac{m_1 + m_2}{2}, \quad \Delta m = m_2 - m_1 \\
\Gamma &= \frac{\Gamma_1 + \Gamma_2}{2}, \quad \Delta \Gamma = \Gamma_2 - \Gamma_1 \\
x &= \frac{\Delta m}{\Gamma}, \quad y = \frac{\Delta \Gamma}{2\Gamma}
\end{align*}
$$
**D oscillation and strong phase**

- short distance is highly suppressed by the GIM mechanism and by the CKM matrix elements within the SM: $x \sim \mathcal{O}(10^{-5})$, $y \sim \mathcal{O}(10^{-7})$
  - NP might manifest in the loop, such as FCNC processes with up-type quark, complementary to those with down quarks (K or B mesons, already studied with observed CPV)

- long distance is dominant:
  - but theoretical uncertainty is large

- Observation of $D_D$ oscillation by CDF and LHCb

$$
\begin{align*}
    y' &\equiv y \cos \delta_{K\pi} - x \sin \delta_{K\pi} \\
    x' &\equiv x \cos \delta_{K\pi} + y \sin \delta_{K\pi}
\end{align*}
$$

- Improving the constraints on the charm mixing parameters is important for testing the SM, such as long-distance effect

- In addition, strong phase is an important ingredient for
  - (over-)constraining the CKM unitarity triangle, which is crucial for searching for new physics
  - extract the mixing parameter $(x,y)$ from $(x', y')$
$D$ mesons from dedicated colliders

**ADONE, FRASCATI ’69-’93**

**SPEAR, SLAC, ’72-’90**

$6 \times 10^{29} \text{ cm}^{-2} \cdot \text{s}^{-1}$

**BEPC, IHEP, ’90-’04**

$5 \times 10^{30} \text{ cm}^{-2} \cdot \text{s}^{-1}$

**CESRc, Cornell, ’04-’08**

$7 \times 10^{31} \text{ cm}^{-2} \cdot \text{s}^{-1}$

**VEPP-4M, Novosibisk, ’02-’12**

$1 \times 10^{30} \text{ cm}^{-2} \cdot \text{s}^{-1}$

**BEPCII, IHEP, ’08-’18(?)**

$1 \times 10^{33} \text{ cm}^{-2} \cdot \text{s}^{-1}$
Beijing Electron Positron Collider-II (BEPCII)

- 2004: started BEPCII/BESIII construction
- 2008: test run
- 2009 - now: BESIII physics run
  - ~60% Physics run
  - ~30% Synchrotron radiation run
The BEPCII Collider

Beam energy: 1.0 – 2.3 GeV

Peak Luminosity:

*Design:* $1 \times 10^{33}$ cm$^{-2}$s$^{-1}$

*Achieved:* $0.7 \times 10^{33}$ cm$^{-2}$s$^{-1}$@3770MeV

Optimum energy: 1.89 GeV

Energy spread: $5.16 \times 10^{-4}$

No. of bunches: 93

Bunch length: 1.5 cm

Total current: 0.91 A

Circumference: 237m

*Beam energy measurement:* Using Compton backscattering technique. Accuracy up to $5 \times 10^{-5}$
Energies of the BEPCII Collider

2 \sim 4.6 \text{ GeV}
The BESIII detector

The new BESIII detector is hermetic for neutral and charged particle with excellent resolution, PID, and large coverage.

Solenoid Magnet: 1 T Super conducting

MDC: small cell & He gas
\( \sigma_{xy} = 130 \, \mu m \)
\( \delta p / p = 0.5\% \) @1GeV
\( dE/dx = 6\% \)

TOF:
\( \sigma_T = 90 \, \text{ps} \) Barrel
\( 110 \, \text{ps} \) Endcap

Muon ID: 8~9 layer RPC
\( \sigma_{R\phi} = 1.4 \, \text{cm} \sim 1.7 \, \text{cm} \)

EMCAL: CsI crystal
\( \Delta E / E = 2.5\% \) @1 GeV
\( \sigma_{\phi,z} = 0.5 \sim 0.7 \, \text{cm} / \sqrt{E} \)

Data Acquisition:
Event rate = 3 kHz
Throughput ~ 50 MB/s

Trigger: Tracks & Showers
Pipelined; Latency = 6.4 \( \mu s \)

NIM A614, 345 (2010)
The BES collaboration

US (6)
- Univ. of Hawaii
- Univ. of Washington
- Carnegie Mellon Univ.
- Univ. of Minnesota
- Univ. of Rochester
- Univ. of Indiana

Europe (13)
- Germany: Univ. of Bochum, Univ. of Giessen, GSI
- Univ. of Johannes Gutenberg Helmholtz Ins. In Mainz
- Russia: JINR Dubna; BINF Novosibirsk
- Italy: Univ. of Torino, Frascati Lab, Ferrara Univ.
- Univ. of Perugia, Univ. of Eastern Piedmont
- Netherlands: KVI/Univ. of Groningen
- Sweden: Uppsula Univ.
- Turkey: Turkey Accelerator Center

Pakistan (2)
- Univ. of Punjab
- COMSAT CIIT

Korea (1)
- Seoul Nat. Univ.

Japan (1)
- Tokyo Univ.

China (29)
- IHEP, CCAST, GUCAS, Shandong Univ., Univ. of Sci. and Tech. of China
- Zhejiang Univ., Huangshan Coll.
- Huazhong Normal Univ., Wuhan Univ.
- Zhengzhou Univ., Henan Normal Univ.
- Peking Univ., Tsinghua Univ.,
- Zhongshan Univ., Nankai Univ., Beihang Univ.
- Shanxi Univ., Sichuan Univ., Univ. of South China
- Hunan Univ., Liaoning Univ.
- Nanjing Univ., Nanjing Normal Univ.
- Guangxi Normal Univ., Guangxi Univ.
- Suzhou Univ., Hangzhou Normal Univ.
- Lanzhou Univ., Henan Sci. and Tech. Univ.

~350 members
52 institutions from 11 countries
## Data samples at BESIII

<table>
<thead>
<tr>
<th></th>
<th>Previous data</th>
<th>BESIII now</th>
<th>Goal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J/\psi$</td>
<td>BESII: 58 M</td>
<td>1.2 B, 20*BESII</td>
<td>10 B</td>
</tr>
<tr>
<td>$\psi(3686)$</td>
<td>CLEO: 28 M</td>
<td>0.5 B, 20*CLEO</td>
<td>3 B</td>
</tr>
<tr>
<td>$\psi(3770)$</td>
<td>CLEO: 0.8 /fb</td>
<td>2.9 /fb, 3.5*CLEO</td>
<td>20 /fb</td>
</tr>
<tr>
<td>Above open charm threshold</td>
<td>CLEO: 0.6/fb @4160MeV</td>
<td>2011: 0.5 /fb @ 4.009 GeV, 0.5 /fb @ 4.36 GeV, 0.6 /fb @ 4.6 GeV, 1.0 /fb @ 4.42 GeV and data for line shape</td>
<td>5-10 /fb</td>
</tr>
<tr>
<td>R Scan</td>
<td>BESII</td>
<td>2012: R @2.23,2.4,2.8,3.4GeV, 25/pb tau threshold, 2014: 0.8 /fb @ 3.85—4.59 GeV</td>
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- world’s largest samples of on-threshold $\psi(3770)$ data and keep increasing in the future
- the aim is to have 20 /fb data
Charm facilities

- Hadron colliders (huge cross-section, energy boost)
  - Tevatron (CDF, D0)
  - LHC (LHCb, CMS, ATLAS)

- $e^+e^-$ Colliders (more kinematic constrains, clean environment, $\sim 100\%$ trigger efficiency)
  - B-factories (Belle, BaBar)
  - Threshold production (CLEOc, BESIII)
    - Quantum Correlations (QC) and CP-tagging are unique
    - Only D meson pairs, no extra CM Energy for pions: clean backgrounds
    - Lots of systematic uncertainties cancellation while applying double tag technique
The decay rate of a correlated state

For a physical process producing $D^0 \bar{D}^0$ such as

$$e^+ e^- \rightarrow \psi'' \rightarrow D^0 \bar{D}^0$$

The $D^0 \bar{D}^0$ pair will be a quantum-correlated state.

The quantum number of $\psi''$ is $J^{PC} = 1^{--}$

The C number of $D^0 \bar{D}^0$ pair in this process is $C = -$

Taking advantage the quantum coherence of $DD$ pairs, BESIII can study the charm physics in an unique way

- strong phase in $D$ decays
- $D$ mixing parameters
- direct CP violation
- ...

Charm tagging at the $\psi(3770)$

$e^+e^- \rightarrow \psi(3770) \rightarrow D\bar{D}$

- Pure $D\bar{D}$ final state, no additional particles ($E_D = E_{beam}$).
- Low multiplicity $\sim 5$-6 charged particles / event
- More kinematical constraints to reconstruct decay channels with $\nu$

- **Single Tag (ST)**
  - Reconstruct one $D$ meson

- **Double Tag (DT)**
  - Tag one $D$ meson in a selected *tag mode*. Study the other $D$ (*signal $D$*).

- **Flavor Tag**
  - Tag the flavor of $D^0$ or $\bar{D}^0$

- **CP tag ($\text{CP}\pm$)**
  - Tag the CP eigenstate: $D_{\text{CP}+}$ or $D_{\text{CP}-}$
Time-integrated decay rates

- No time dependent information at Charm threshold
- Anti-symmetric wavefunction:
  \[ \Gamma_{ij}^2 = \left| \langle i|D^0\rangle\langle j|\bar{D}^0\rangle - \langle j|D^0\rangle\langle i|\bar{D}^0\rangle \right|^2 \]
- Double tag rates:
  \[ A_i^2 A_j^2 \left[ 1 + r_i^2 r_j^2 - 2r_i r_j \cos(\delta_i + \delta_j) \right] \]
- CP tag: \( r=1, \delta=0 \) or \( \pi \); \( l^\pm \) tag: \( r=0 \)
- Single and Double tag rates

\[ z_f \equiv 2 \cos \delta_f, r_f \equiv \frac{A_{DCS}}{A_{CF}}, R_M \approx \frac{x^2 + y^2}{2} \]

<table>
<thead>
<tr>
<th>C-odd</th>
<th>( f )</th>
<th>( \bar{f} )</th>
<th>( l^+ )</th>
<th>( l )</th>
<th>( CP^+ )</th>
<th>( CP^- )</th>
</tr>
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<tbody>
<tr>
<td>( f )</td>
<td>( R_M [1 + r_f^2 (2 - z_f^2) + r_f^4] )</td>
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<tr>
<td>( \bar{f} )</td>
<td>( 1 + r_f^2 (2 - z_f^2) + r_f^4 )</td>
<td>( R_M [1 + r_f^2 (2 - z_f^2) + r_f^4] )</td>
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<tr>
<td>( l^+ )</td>
<td>( r_f^2 )</td>
<td>( l )</td>
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<td></td>
<td>( R_M )</td>
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</tr>
<tr>
<td>( l )</td>
<td>( l )</td>
<td>( r_f^2 )</td>
<td></td>
<td>( R_M )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( CP^+ )</td>
<td>( 1 + r_f (r_f + z_f) )</td>
<td>( 1 + r_f (r_f + z_f) )</td>
<td>( l )</td>
<td>( 1 )</td>
<td>( 0 )</td>
<td></td>
</tr>
<tr>
<td>( CP^- )</td>
<td>( 1 + r_f (r_f - z_f) )</td>
<td>( 1 + r_f (r_f - z_f) )</td>
<td>( l )</td>
<td>( 1 )</td>
<td>( 4 )</td>
<td>( 0 )</td>
</tr>
</tbody>
</table>

Single Tag: \( 1 + r_f^2 - r_f z_f (A - y) \)
Analysis techniques

Quantum Correlated topics

✦ Mixing \((x^2+y^2): D\bar{D} \rightarrow (K^-\ell^+\nu)^2, (K^-\pi^+)^2\)
✦ Strong phase \(c_\delta\): Double Tag Events: \(K^-\pi^+\) vs CP ±
✦ Charm Mixing \((y_{CP})\): Flavor Tag vs CP ±
✦ DCS: Wrong sign decays \(K^-\pi^+\) vs \(K^-\ell^+\nu\)
✦ Strong phase \(c_i,s_i\) (Dalitz): \(K_S\pi^+\pi^-\) vs CP ±; \(K_S\pi^+\pi^-\) vs Flavor Tag; \(K_S\pi^+\pi^-\) vs \(K_{S,L}\pi^+\pi^-\)
✦ Typical Kinematic variables for full reconstruction
  ✦ Energy difference & Beam Constrained mass
  \[ \Delta E = E_D - E_{Beam} \]
  \[ M_{BC} = \sqrt{E^2_{Beam} - \vec{p}^2_D} \]

Global fit method

✦ Combined analysis to extract mixing parameters, DCS, strong phase plus charm hadronic branching fractions
QC inputs for Charm Physics

- Precision CKM test
- Charm Mixing & CP violation

- inputs from Quantum Correlated (QC)
  \( \psi(3770) \rightarrow D \bar{D} \) decays
  - (Averaged) Strong phase difference: \( \delta_D \)
  - Coherent factors: \( R_D \)
  - (Averaged) Strong phase in Dalitz bins: \( c_i, s_i \)
- \( B \) factories, LHCb, Super \( B \) factories are the customers
Strong phase $\delta$ and $\gamma/\phi_3$ in the CKM unitarity triangle

- $D$ hadronic parameters for a final state $f$: \[ \frac{A(D^0 \to f)}{A(D^0 \to \bar{f})} \equiv -r_D e^{-i\delta_D} \]
- Charm mixing parameters: $x = \frac{\Delta M}{\Gamma}$, $y = \frac{\Delta \Gamma}{2\Gamma}$
  - Time-dependent WS $D^0 \to K^+ \pi^-$ rate ⇒ $y' = y \cos \delta_{K\pi} - x \sin \delta_{K\pi} = (4.8 \pm 1.0) \times 10^{-3}$ (LHCb2013)
  - $\delta_{K\pi}$: QC measurements from Charm factory
- $\gamma/\phi_3$ measurements from $B \to D^0 K$
  - $b \to u : \gamma/\phi_3 = \arg V^*_{ub}$
  - most sensitive method to constrain $\gamma/\phi_3$ at present
  - GLW method (Gronau & London, PLB253, 483 (1991); Gronau & Wyler, PLB265, 172 (1991))
  - ADS method (Atwood, Dunetz & Soni, PRL78, 3257 (1997); PRD63, 036005 (2001))
  - GGSZ (Dalitz) method (Giri, Grossman, Soffer & Zupan, PRD68, 054018 (2003))
- GLW and ADS methods in $B \to D^0 K$
  - $D^0$ to doubly Cabibbo suppressed decays $K^+\pi^-$, $K^+\pi^-\pi^0$
  - Decay rates:
    \[ \Gamma(B^\pm \to (f)_D K^\pm) \propto r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D \pm \phi_3) \]
- $r_D$, $\delta_D$: QC measurements from Charm factory
- $(r_B, \delta_B, \phi_3)$ 3 unknowns, 4 measurements
\(c_i, s_i\) for \(\gamma/\phi_3\) measurements

- **GGSZ (Dalitz) method in** \(B \to D^0 K\)
  - Most powerful method nowadays
  - \(D^0\) to 3-body decays \(K_S\pi^+\pi^-, K_SK^+K^-,\ldots\)
  - Partition the Dalitz plot to 2k bins
  - \(B\) Signal yields in \(i^{th}\) Dalitz bin

\[
\begin{align*}
N_{i}^{\pm} &= h_B [K_{i}^{\pm} + r_B^2 K_{i}^{\mp} + 2\sqrt{K_i K_{-i}} (x_i c_i \pm y_i s_i)]
\end{align*}
\]

- Averaged phases in each bin: \(c_i, s_i\) (Giri et. al. PRD68, 054018 (2003))

\[
\begin{align*}
A_D(s_{12}, s_{13}) &= A_{12,13} e^{i\delta_{12,13}} \\
&= A(D^0 \to K_S^0 (p_1)\pi^- (p_2)\pi^+ (p_3)) \\
&= A(\bar{D}^0 \to K_S^0 (p_1)\pi^+ (p_2)\pi^- (p_3)) \\
&= \int dp A_{12,13} A_{13,12} \cos (\delta_{12,13} - \delta_{13,12})
\end{align*}
\]

\[
\begin{align*}
&c_i = \int dp A_{12,13} A_{13,12} \cos (\delta_{12,13} - \delta_{13,12}) \\
s_i = \int dp A_{12,13} A_{13,12} \sin (\delta_{12,13} - \delta_{13,12}) \\
&T_i = \int dp A_{12,13}^2 \\
&c_i, s_i : \text{weighted average of } \cos(\Delta\delta_D) \text{ and } \sin(\Delta\delta_D) \text{ respectively where } \Delta\delta_D \\
&\text{is the difference between phase of } D^0 \text{ and } \bar{D}^0
\]
CKM measurements: current status

\[
\begin{align*}
\alpha/\phi_2 &= \left(85.4^{+4.0}_{-3.9}\right) \\
\beta/\phi_1 &= \left(21.38^{+0.79}_{-0.77}\right)^\circ \\
\gamma/\phi_3 &= \left(68^{+8.0}_{-8.5}\right)
\end{align*}
\]

2013 CKMfitter (direct measurements)

- \(\gamma\) is the least precisely measured angle.
- Precision can still be improved by BESIII with cooperation from BaBar, Belle, LHCb.
- Precise measurement of \(\gamma\) is needed to test the consistency of the CKM UT.
- Any difference between Tree measurements to Loop measurements might be a sign of New Physics in the flavor sector.

Example of \(\gamma\) measurements from Dalitz

- **Belle Model-Dependent Dalitz** [Phys. Rev. D 81, 112002 (2010)]
  \[78.4^{+10.8}_{-11.6} (stat) \pm 3.6 (syst) \pm 8.9 (Model)\]

- **Belle Model-Independent Dalitz** [Phys. Rev. D 85, 112014 (2012)]
  \[77.3^{+15.1}_{-14.9} (stat) \pm 4.2 (syst) \pm 4.3 (c_i/s_i)\]

Currently statistically limited, but soon systematically limited
Strong Phase $\delta_{K\pi}$

Quantum correlation $\rightarrow$ Interference $\rightarrow$ access strong phase!

$\langle K\pi | D_{CP\pm} \rangle = (\langle K\pi | D^{0} \rangle \pm \langle K\pi | \bar{D}^{0} \rangle) / \sqrt{2} \Rightarrow \sqrt{2} A_{CP\pm} = A_{K\pi} \pm \overline{A_{K\pi}}$

$2r_{K\pi} \cdot \cos \delta_{K\pi} \approx A_{CP-K\pi} \equiv \frac{|A_{CP-}|^2 - |A_{CP+}|^2}{|A_{CP-}|^2 + |A_{CP+}|^2} = \frac{Br(D_{CP^{-}\rightarrow K\pi}) - Br(D_{CP^{+}\rightarrow K\pi})}{Br(D_{CP^{-}\rightarrow K\pi}) + Br(D_{CP^{+}\rightarrow K\pi})}$

Measuring $\delta_{K\pi}$ from rate differences if using external $r_{K\pi}$

Reconstructed modes:
- Flavor tags: $K^{-}\pi^{+}$, $K^{+}\pi$
- CP+ tags (5 modes): $K^{-}K^{+}$, $\pi^{+}\pi^{-}$, $K_{S}^{0}\pi^{0}\pi^{0}$, $\pi^{0}\pi^{0}$, $\rho^{0}\pi^{0}$
- CP- tags (3 modes): $K_{S}^{0}\pi^{0}$, $K_{S}^{0}\eta$, $K_{S}^{0}\omega$
Strong Phase $\delta_{K\pi}$

**Signal reconstruction:**
- Single Tag (ST): CP tags
- Double Tag (DT): $K\pi +$ CP Tag
- Kinematic variable: Beam Constrained Mass ($M_{BC}$)
- Singal shape: $\sigma \otimes$ MC-truth
- Background shape: ARGUS function

$$Br(D_{CP\pm} \to K\pi) = \frac{n_{K\pi,CP\pm}}{n_{CP\pm}} \cdot \frac{\varepsilon_{CP\pm}}{\varepsilon_{K\pi,CP\pm}}$$

- $n_{K\pi,CP\pm}$ and $n_{CP\pm}$ are event yields for DT and ST from $M_{BC}$ fit
- $\varepsilon_{K\pi,CP\pm}$ and $\varepsilon_{CP\pm}$ are detection efficiencies of DT and ST from MC simulation
- Most systematics cancelled for ratio $\varepsilon_{CP\pm}/\varepsilon_{K\pi,CP\pm}$

**BESIII results:**

$$A_{CP \to K\pi} = (12.7 \pm 1.3 \pm 0.7) \times 10^{-2}$$
\[ \delta_{K\pi} \text{ in } D \to K\pi \]

- If we don’t ignore the mixing effect
- \[ 2r_{K\pi} \cos \delta_{K\pi} + y = (1 + R_{WS}) \cdot A_{CP\to K\pi} \]
- \[ R_{WS} = \frac{\Gamma(D^0\to K^+\pi^-)}{\Gamma(D^0\to K^-\pi^+)} = r_{K\pi}^2 + r_{K\pi} y + \frac{(x^2 + y^2)}{2} \]
- External inputs from HFAG2013 and PDG
  - \[ r_{K\pi}^2 = 0.347 \pm 0.006\%, \]
  - \[ y = 0.66 \pm 0.09\%, \]
  - \[ R_{WS} = 0.380 \pm 0.005\% \]


\[
\begin{align*}
\cos \delta_{K\pi} &= 0.81^{+0.22}_{-0.18}^{+0.07}_{-0.05} \\
\cos \delta_{K\pi} &= 1.15^{+0.19+0.00}_{-0.17-0.08} \quad \text{(globalfit)}
\end{align*}
\]

BESIII results:
\[ \cos \delta_{K\pi} = 1.02 \pm 0.11 \pm 0.06 \pm 0.01 \]

- The third error is due to the input parameters
- The statistical errors dominant the precision
- World best precision
- In 20 \( fb \) BESIII data, precision of \( \cos \delta_{K\pi} \) will reach \( \sim 0.05 \)
  \[ \rightarrow \text{updated LHCb: stat. } \sigma(\phi_3) \text{ will be } \sim 1.3^\circ \quad \text{EPJC73, 2373(2013)} \]

We need super-charm factory for a 1 \( ab \) data, which can improve \( \sigma(\phi_3) \) due to strong phase to \( \sim 1^\circ \)!
$c_i, s_i \text{ in } D^0 \rightarrow K_{S,L} \pi^+ \pi^-$ Dalitz analysis

We can calculate $c_i$ and $s_i$ from double tags of $D^0 \rightarrow K_S \pi^+ \pi^-$ vs $D^0 \rightarrow (K_{S,L} \pi^+ \pi^- \text{ or CP eigenstates})$

A relationship can be shown between Dalitz bin yields and $c_i$ and $s_i$ (in backup slides).

Only $c_i, s_i$ from $K_S \pi^+ \pi^-$ is used to calculate $\gamma$.
However adding in $D^0 \rightarrow K_L \pi^+ \pi^-$ we can calculate $c'_i, s'_i$ and use how they relate to $c_i, s_i$ to further constrain our results in a Global fit.

(BESIII: 2.9 fb$^{-1}$)
Binning in Dalitz plots

Result of splitting the Dalitz phase space into 8 equally spaced phase bins based on the BaBar 2008 Model.

Starting with the equally spaced bins, bins are adjusted to optimize the sensitivity to $\gamma$. A secondary adjustment smooths binned areas smaller than detector resolution.

Similar to the “optimal binning” except the expected background is taken into account before optimizing for $\gamma$ sensitivity.
$K^0_s \pi^+ \pi^-$ Dalitz plots vs $CP$ Modes

- Data is using the full 2.9 fb$^{-1}$ $\psi(3770)$ dataset
- Results presented here will be using Optimal Binning scheme.

BESIII preliminary
Preliminary Data Results

- Still statistical limited.
- Only statistical errors are listed.
- Consistent agreement with CLEO-c measurements, but superior in statistical errors.

Based on the BESIII results, we expect a reduction in the \((c_i, s_i)\) contribution to the uncertainty in \(\gamma/\phi_3\) of \(\sim 40\%\) \((\sim 80\%\ in\ 20\ /fb\ data)\).

\[
77.3^{+15.1}_{-14.9}\ (stat) \pm 4.2\ (syst) \pm 4.3(c_i/s_i) \pm 2.5 (0.9) (c_i/s_i)
\]

Crucial inputs for the future analysis carried out in the LHCb and super-B experiment (stat. sensitivity reaches 1\(\sim 2\°\)).
We measure the $y_{CP}$ using CP-tagged semi-leptonic D decays, which allows to access CP asymmetry in mixing and decays.

\[ \begin{align*}
\text{Single Tag decay rate (CP tags)} & : \quad \Gamma_{CP} \propto 2|A_{CP}|^2 (1 \mp y) \\
\text{Double Tag decay rate (Flavor tags + CP tags)} & : \quad \Gamma_{l;CP} \propto |A_l|^2 |A_{CP}|^2 \\
\text{Neglect term } y^2 \text{ or higher order} & : \quad y_{CP} \approx \frac{1}{4} \left( \frac{\Gamma_{l;CP} + \Gamma_{CP}}{\Gamma_{l;CP} - \Gamma_{CP}} \right)
\end{align*} \]

Reconstructed modes:
- Flavor tags: $K\nu_e, K\nu_\mu$
- CP+ tags (3 modes): $K^-K^+, \pi^+\pi^-, K_S^0\pi^0\pi^0$
- CP- tags (3 modes): $K_S^0\pi^0, K_S^0\eta, K_S^0\omega$
Signal reconstruction:
-
- **Single tag yields extraction:**
  - Single shape: $\sigma \otimes$MC-truth
  - Background: ARGUS function
  - Kinematic variable: $M_{BC}$

- **Double tag yields extraction:**
  - Singal shape: $\sigma \otimes$MC-truth
  - Background: Polynomial
  - $K\pi\pi^0$ background shape from data
  - Kinematic variable:
    \[ U_{\text{miss}} = E_{\text{miss}} - |\vec{P}_{\text{miss}}| \approx 0 \text{ for signals} \]

- **BESIII preliminary results:**
  \[ y_{CP} = (-1.6 \pm 1.3 \pm 0.6)\% \]

- Most precise measurement with QC charm mesons
- In the limit of no CP violation: $y_{CP} = y$

**THIS WORK**

\[ \text{(BESIII: 2.9 fb}^{-1}) \]

**y_{CP} measurement**

**Double Tags**

**Single Tags**
Other Quantum Correlation work

Coherence factors: feed into CKM $\gamma/\phi_3$ with B’s

\[
\begin{align*}
K^-\pi^+\pi^0 & \\
\pi^+\pi^-\pi^0 & \\
\pi^+\pi^-\pi^0
\end{align*}
\]

likely to pursue both model-independent analyses, and also detailed Dalitz analyses

+ other modes e.g., $K^-\pi^+\pi^+\pi^-$ & $K_S K^+\pi^-$ have been done by CLEO-c

BESIII will publish these works in the future with best precisions.
BESIII runs in the future

**Future Runs:** (no particular order)

- lower-energy R scan
- $D_s^*D_s$ data @ 4170 MeV
- More $\psi$(3770), $J/\psi$, $\psi'$
- More “XYZ”

Easy to feed the BESIII spectrometer in many years!
High Intensity Electron Positron Accelerator Facility (HIEPAF)

- Providing peak luminosity about $1 \times 10^{35} \text{ cm}^{-2}\text{s}^{-1}$ at 4 GeV for physics at tau charm sector, covering $E_{\text{cm}} = 2$-7 GeV.
HIEPAF Study Time Line

<table>
<thead>
<tr>
<th>2013</th>
<th>2014</th>
<th>2015</th>
<th>2016</th>
<th>2017</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>Q2</td>
<td>Q3</td>
<td>Q4</td>
<td>Q1</td>
</tr>
<tr>
<td>Q2</td>
<td>Q3</td>
<td>Q4</td>
<td>Q1</td>
<td>Q2</td>
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<td>Q3</td>
<td>Q4</td>
<td>Q1</td>
<td>Q2</td>
<td>Q3</td>
</tr>
<tr>
<td>Q4</td>
<td>Q1</td>
<td>Q2</td>
<td>Q3</td>
<td>Q4</td>
</tr>
</tbody>
</table>

- **Kick-off**
  - collaboration forming

- **Workshops**
  - Feasibility study
  - Review

- **CDR, R&D → TDR?**
Direct CP violation in D-$\rightarrow$hh

- World measurements of
  \[ \Delta A_{CP} = A_{CP}(D \rightarrow KK) - A_{CP}(D \rightarrow \pi\pi) \]

HFAG world-average:
- \[ \Delta A_{CP} = (-0.33 \pm 0.12)\% \]

the individual asymmetries:
- \[ A_{KK} = -0.16 \pm 0.20 \]
- \[ A_{\pi\pi} = +0.16 \pm 0.21 \]

- In New Physics: CPV up to $\sim$1%;
- If CPV $\sim$1% were observed, is it NP or hadronic enhancement of SM?
- **Strategy**: analyze many channels to elucidate source of CPV.
- 50 ab$^{-1}$ BELLE-II data can reach to sensitivity of $10^{-3}$$\sim$$10^{-4}$ for different channels
- Measurement of direct CPV at threshold at HIEPAF with 1 ab$^{-1}$ data, sensitivity: $10^{-3}$$\sim$$10^{-4}$
- clean background and better systematic control in threshold production would be complementary to the future $B$ factory results

No CPV observed yet in charm sector.
Determination of the $\gamma$ angle in the CKM matrix

The cleanest way to extract $\gamma$ is from $B \rightarrow DK$ decays:

- current uncertainty $\sigma(\gamma) \sim 8^\circ$
- however, theoretical error: $10^{-6}$ (!)
- over-constrain the Unitarity Triangle

Information of $D$ decay strong phase is needed, which can only be accessed through quantum coherence of $DD$ production at threshold:

- ADS method: use $D$ doubly Cabibbo-suppressed decays, e.g. $D^0 \rightarrow K^+\pi^-$
  - With 1 ab$^{-1}$ at HIEPAF: $\sigma(\cos\delta_{K\pi}) \sim 0.007$; $\sigma(\delta_{K\pi}) \sim 2^\circ$
  - With this data, upgraded LHCb: $\sigma(\gamma) \sim 1.3^\circ$

- GGSZ method: use Dalitz plot analysis of 3-body $D^0$ decays, e.g. $K_s \pi^+ \pi^-$; high statistics; need precise Dalitz model
  - Belle results from GSSZ method in 2012: $\gamma = (77 \pm 15 \pm 4 \pm 4)^\circ$
  - HIEPAF would reduce the contribution of $D$ Dalitz model to a comfort level of $\sim 0.1^\circ$, since expected precision from future B projects would be $\sim 2^\circ$.
**D–D mixing at HIEPAF**

- **D⁰ – D⁰ mixing rate at threshold**
  \[
  R_M = \frac{x^2 + y^2}{2} = \frac{N[(K^\pm \pi^\mp)]}{N[(K^\mp \pi^\mp)]}
  \]

  with 1 ab⁻¹ data:
  - \( R_M = (x^2 + y^2)/2 \sim 10^{-5} \) in Kπ and Keν channels
  - Probe \( y \): \( \Delta y_{CP} < 0.1\% \)

While at LHCb by 2015 assuming 5 fb⁻¹ @ 14 TeV:
  - \( (x', y') \pm (0.004, 0.08)\% \) with WS \( D^0 \to K\pi \)
  - \( D^0-D^0 \) WS rate asymmetry (like \( |q/p| \)) ±1%
  - \( y_{CP}, A_{\Gamma} \pm 0.02\% \) in \( D^0 \to K^-K^+ \) and \( D^0 \to \pi^-\pi^+ \) decays
  - \( (x, y) \pm (0.045, 0.030)\% \) using \( D^0 \to K_S\pi\pi \)

- **Implication of strong phase**
  - phase to relate \( (x', y') \) with \( (x, y) \)
  - upgraded LHCb/BELLE-II expect statistical error
    \[
    \sigma(x, y) \sim 0.2 \times 10^{-3}, \sigma(r_{CP}) \sim 1\%, \sigma(\alpha_{CP}) \sim 0.7^\circ
    \]
  - 1 ab⁻¹ \( D \) data at HIEPAF would reach to the sub-level of the above stat. errors

*charm mixing is established!*

*arXiv:1209.0172*
Asymmetric beam? charm mixing at $\Psi(4040)$?

- Flavor tagging ($D^0 \rightarrow K^{-} l^{+} \nu$)
- Time-dependent measurement available

\[
\mathcal{A}(\Delta t) = \frac{\Gamma(\Delta t) - \Gamma(\Delta t)}{\Gamma(\Delta t) + \Gamma(\Delta t)}
\]

- Require good $\Delta z/\Delta t$ resolution (SVT)

Considering $\Psi(4040) \rightarrow DD^{*} \rightarrow DD \gamma$, $DD$ pairs are in $C$-even states and charm mixing contribution is doubled compared with time-dependent (uncorrelated) case.

**CPV and D mixing reach**

![Graphical representation of CPV and D mixing reach]

courtesy by Neri&Rama

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\Psi(3770)$</th>
<th>$\Psi(4040)$</th>
<th>LHCb</th>
<th>Belle-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x(%)$</td>
<td>0.02-0.05</td>
<td>0.03</td>
<td>0.015</td>
<td>0.08</td>
</tr>
<tr>
<td>$y(%)$</td>
<td>0.02-0.03</td>
<td>0.03</td>
<td>0.010</td>
<td>0.04</td>
</tr>
<tr>
<td>$</td>
<td>q/p</td>
<td>(%)$</td>
<td>2-5</td>
<td>0.9</td>
</tr>
<tr>
<td>$\arg(q/p)(\degree)$</td>
<td>2-3</td>
<td>0.8</td>
<td>3</td>
<td>2.6</td>
</tr>
</tbody>
</table>

3 ab$^{-1}$ data @$\Psi(3770)$: asymmetric $\gamma\beta$=0.2~0.6

3 ab$^{-1}$ data @$\Psi(4040)$

50 fb$^{-1}$ data at upgrade LHCb

50 ab$^{-1}$ at BELLE-II
Summary

✦ BESIII has started to be involved in the world campaign on identifying the $D\bar{D}$ oscillation, searching for the $CPV$ in charm and over-constraining the CKM unitarity triangles.
  ‣ We provide unique data on strong phases.
  ‣ Many more QC analyses are undergoing.
  ‣ The simultaneous fitting package has been developed at BESIII. The package will be applied on measuring strong phases and mixing parameters.
✦ Possible extensions: C-even (C=+1) Quantum Correlated Analysis $D^0\bar{D}^0\gamma$, $D^0\bar{D}^0\pi^0$ from higher-energy data?
✦ Project of a super-tau-charm factory in China is being proposed
Backup
Unitarity Triangle

\[ V^+V = I \Rightarrow \begin{pmatrix} V_{ud}^* & V_{cd}^* & V_{td}^* \\ V_{us}^* & V_{cs}^* & V_{ts}^* \\ V_{ub}^* & V_{cb}^* & V_{tb}^* \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

\[ V_{ub}V_{ud} + V_{cb}V_{cd} + V_{tb}V_{td} = 0 \]

\( \text{(B system)} \)
\[
\phi_3 \text{ from } B^- \rightarrow D^0 K^-
\]

\[
\frac{A(B^+ \rightarrow D^0 K^+)}{A(B^+ \rightarrow D^0 K^+)} = r_B e^{i(\delta_B + \phi_3)}
\]

★ No hadronic uncertainty
★ Three methods for exploiting interference (choice of D^0 decay modes):
  ✤ Gronau, London, Wyler (GLW): Use CP eigenstates of D(\*)^0 decay, e.g. D^0 \rightarrow K_s \pi^0, D^0 \rightarrow \pi^+ \pi^-
  ✤ Atwood, Dunietz, Soni (ADS): Use doubly Cabibbo-suppressed decays, e.g. D^0 \rightarrow K^+ \pi^-
  ✤ Giri, Grossman, Soffer, Zupan (GGSZ): Use Dalitz plot analysis of 3-body D^0 decays, e.g. K_s \pi^+ \pi^-; high statistics; need precise Dalitz model
★ Problem: statistics
Gronau-Wyler original method

\[ A(B^+ \rightarrow D^0K^+) = \sqrt{2} A(B^- \rightarrow D^0K^-) \]

Theoretically clean
Experimentally challenging
Hadronic $D$ decay modes: hard for $D$ flavor tagging
Semi-leptonic $D$ decays: Background too high
$CP$ eigenstate decays of $D$: small Branching ratio
Atwood-Dunietz-Soni Method

- Use interference between $B^+ \to DK^+$ and $B^+ \to \overline{D}K^+$ follows by $D (\overline{D}) \to f$
- To get a common final state $f$, we need
  - Double Cabibbo Suppression (DCS): $f = K^+ \pi^-, K^+ K^-$
  - $K - \bar{K}$ mixing: $f = K_S \pi^0, K_S \pi^+ \pi^-$

- $D$ hadronic parameters:
  \[
  \frac{A_{DCS}(D^0 \to f)}{A_{CA}(D^0 \to f)} \equiv r_D e^{i\delta_D}
  \]

- Decay rates:
  \[
  \Gamma(B^\pm \to (f)_D K^\pm) \propto r_B^2 + r_D^2 + 2r_D r_B \cos(\delta_B + \delta_D \pm \phi_3)
  \]

- $r_D, \delta_D$: measured from Charm factory (see next slides)

- $(r_B, \delta_B, \phi_3)$: 3 unknowns, 4 measurements $\Rightarrow \phi_3$
Calculation of $c_i$, $c'_i$, $s_i$, $s'_i$

From the CP tag modes, we are able to find $c_i$ and $c'_i$:

\[
M_i = \frac{S_{\pm}}{2S_f} (K_i \pm 2c_i \sqrt{K_i K_{\bar{i}}} + K_{\bar{i}}) \quad (CP, K_S^0 \pi^+ \pi^-)
\]

\[
M'_i = \frac{S_{\pm}}{2S_f} (K'_i \pm 2c'_i \sqrt{K'_i K'_{\bar{i}}} + K'_{\bar{i}}) \quad (CP, K_L^0 \pi^+ \pi^-)
\]

$M_i$ yields in each bin of Dalitz plot for CP even(odd) modes.
$S_{\pm}(S_-)$, number of single tags for CP even(odd) modes.
$K_i(K_{\bar{i}})$, yields in each bin of Dalitz plot in flavor modes.

From the Double Dalitz modes, we are able to find $c_i$, $c'_i$, $s_i$, $s'_i$:

\[
M_{i,j} = \frac{N_{D,D}}{2S_f^2} (K_i K_j + K_{\bar{i}} K_{\bar{j}} - 2\sqrt{K_i K_{\bar{i}} K_j K_{\bar{j}}} (c_i c_j + s_i s_j)) \quad (K_S^0 \pi^+ \pi^-, K_S^0 \pi^+ \pi^-)
\]

\[
M'_{i,j} = \frac{N_{D,D}}{2S_f} (K_i K'_j + K_{\bar{i}} K'_{\bar{j}} + 2\sqrt{K_i K'_j K_{\bar{i}} K'_{\bar{j}}} (c_i c'_j + s_i s'_j)) \quad (K_S^0 \pi^+ \pi^-, K_L^0 \pi^+ \pi^-)
\]

$M_{i,j}$ yields in each $i^{th}$ bin of the first Dalitz plot and the $j^{th}$ bin for the second Dalitz plot.
$S_f$, number of single tags for flavor modes.
$K_i(K_{\bar{i}})$, yields in each bin of Dalitz plot in flavor modes.
updated $\delta$ in $D \rightarrow K\pi$ (CLEO-c: 818 pb$^{-1}$)
updated $\delta$ in $D \rightarrow K\pi$ (CLEO-c: 818 pb$^{-1}$)  

PRD86,112001(2012)

- Fit has 51 parameters:
  - $N_{DD} + 21$ BFs + 24 amplitudes & phases for $K_S\pi^+\pi^- + 5 K\pi$ and mixing parameters
- Statistical errors on $y$ and $r_{K\pi}\cos\delta_{K\pi}$ 3x smaller than 2008 analysis (standard fit)
  - First direct measurements of $r_{K\pi}^2$ and $\sin\delta_{K\pi}$
  - When used to average $y$ and $y'$, improves overall precision by 12%.
- $\sin\delta_{K\pi}$ sign ambiguity in standard fit
- All measurements are statistics-limited

<table>
<thead>
<tr>
<th>Parameter</th>
<th>HFAG 2010 CLEO 2008</th>
<th>Fit: no ext. meas. (standard)</th>
<th>Fit: w/ ext. $y$, $x$, $y'$ (extended)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$ ($10^{-2}$)</td>
<td>$0.79 \pm 0.13$</td>
<td>$4.2 \pm 2.0 \pm 1.0$</td>
<td>$0.636 \pm 0.114$</td>
</tr>
<tr>
<td>$\chi^2$ ($10^{-3}$)</td>
<td>$0.037 \pm 0.024$</td>
<td>$0.6 \pm 2.3 \pm 1.1$</td>
<td>$0.022 \pm 0.023$</td>
</tr>
<tr>
<td>$r_{K\pi}^2$ ($10^{-3}$)</td>
<td>$3.32 \pm 0.08$</td>
<td>$5.33 \pm 1.07 \pm 0.45$</td>
<td>$3.33 \pm 0.08$</td>
</tr>
<tr>
<td>$\cos\delta_{K\pi}$</td>
<td>$1.10 \pm 0.36$</td>
<td>$0.81 ^{+0.22}<em>{-0.18} ^{+0.07}</em>{-0.05}$</td>
<td>$1.15 ^{+0.19}_{-0.17} ^{+0.0} _{-0.08}$</td>
</tr>
<tr>
<td>$\sin\delta_{K\pi}$</td>
<td>---</td>
<td>$-0.01 \pm 0.41 \pm 0.04$</td>
<td>$0.56 ^{+0.32}<em>{-0.31} ^{+0.21}</em>{-0.20}$</td>
</tr>
<tr>
<td>$\delta_{K\gamma}$ ($^{\circ}$) [derived]</td>
<td>$22^{+11}<em>{-12} ^{+9}</em>{-11}$</td>
<td>$10^{+28}<em>{-53} ^{+13}</em>{-0}$</td>
<td>$18^{+11}_{-17}$</td>
</tr>
</tbody>
</table>

2.5σ diff. due to fluctuations in $r^2$ and $y$, correlated with $\cos\delta_{K\pi}$

Average of $y$ and $y' = y \cos\delta_{K\pi} - x \sin\delta_{K\pi}$
\[ D^0 \rightarrow K_S K^+ \pi^- \text{ and } D^0 \rightarrow K_S K^- \pi^+ \text{ (CLEO-c: 818 pb}^{-1}) \]

**Non-flavored, non-self-conjugate modes (Singly-Cabibbo-suppressed)**

**Motivations**
- Coherence factors & Strong phases
  ⇒ input for \( \gamma/\phi_3 \) measurements
- “Generic” Dalitz analysis
  ⇒ resonant substructures, BFs of two modes
  - CLEOIII continuum charm w/ D* tag
  - CLEO-c with flavor tags
- Time-dependent study ⇒ Charm mixing & CPV

**Formalism (Dalitz case)**

\[
\Gamma(B^+ \rightarrow D(K^+ \pi^-)K^+) \propto (r_B)^2 + (r_D^K\pi)^2 + 2r_Br_D^K\pi \cos(\delta_B + \delta_D^K\pi + \gamma),
\]

\[
\Gamma(B^+ \rightarrow D(K_S^0 K^+ \pi^-)K^+) \propto (r_B)^2 + (r_D^{K_S^0 K^+\pi^-})^2 + 2r_Br_D^{K_S^0 K^+\pi^-} \cos(\delta_B + \delta_D^{K_S^0 K^+\pi^-} + \gamma),
\]

\[
R_{K_S^0 K^+ \pi^-} = \frac{A_{K_S^0 K^+ \pi^-} \langle m^2_{K_S^0 K^+ \pi^-} \rangle A_{K_S^0 K^+ \pi^-} \langle m^2_{K_S^0 K^+ \pi^-} \rangle}{A_{K_S^- K^+ \pi^-} A_{K_S^- K^+ \pi^-}}
\]

- Coherence factor \( R \) can be \([0, 1]\)
  - One state dominant ⇒ \( R \sim 1 \)
$D^0 \rightarrow K_S K^+ \pi^-$ and $D^0 \rightarrow K_S K^- \pi^+$ (CLEO-c: 818 pb$^{-1}$)

- **PRD85, 092016 (2012)**
- **Coherence Factor Analysis:**
  - Data sample: CLEO-c with CP-tags
- **Tagged modes:**
  - Flavor tags: $K^- \pi^+$, $K^- \pi^+ \pi^0$, $K^- \pi^+ \pi^+ \pi^-$
  - CP+ tags: $K^- K^+$, $\pi^+ \pi^-$, $K_S^0 \pi^0 \pi^0$, $K_L^0 \pi^0$, $K_L^0 \eta$, $K_L^0 \omega$, $K_L^0 \eta'$
  - CP- tags: $K_S^0 \pi^0$, $K_S^0 \eta$, $K_S^0 \omega$, $K_S^0 \eta'$, $K_L^0 \pi^0 \pi^0$
  - Mixed CP tags: $K_S^0 \pi^+ \pi^-$, $K_L^0 \pi^+ \pi^-$
- **Results in full Dalitz plot region:**
  - $R = 0.73 \pm 0.08$
  - $\delta = (8.3 \pm 15.2)^\circ$
- **Results for a restricted region (±100 MeV around $K^*(892)^+$):**
  - $R = 1.00 \pm 0.16$
  - $\delta = (26.5 \pm 15.8)^\circ$
$D \to K\pi\pi^0$ and $D \to K3\pi$ (CLEO-c: 818 pb$^{-1}$)

- **Motivation**
  - Coherence factors & Strong phases $\Rightarrow$ input for $\gamma/\phi_3$ measurements

- **Formalism** (Dalitz case: similar to $D^0 \to K_SK\pi$)
  \[
  R_{K\pi\pi^0} e^{-i\delta_D^{K\pi\pi^0}} = \frac{\int A_{K^-\pi^+\pi^0}(x) A_{K^+\pi^-\pi^0}(x) dx}{A_{K^-\pi^+\pi^0} A_{K^+\pi^-\pi^0}} \quad \delta_D^{K\pi\pi^0} = \frac{A_{K^-\pi^+\pi^0}}{A_{K^-\pi^+\pi^0}}
  \]

- **Tagged modes:**
  - Flavor tags: $K^{-}\pi^+, K^{-}\pi^+\pi^0, K^{-}\pi^+\pi^+\pi^-$
  - CP+ tags: $K^-K^+, \pi^+\pi^-, K_S^0\pi^0\pi^0, K_L^0\pi^0, K_L^0\omega$
  - CP- tags: $K_S^0\pi^0, K_S^0\eta, K_S^0\omega, K_S^0\eta', K_S^0\phi$

- **First determination of $R_{K\pi\pi^0}, R_{K3\pi}, \delta_D^{K\pi\pi^0}, \delta_D^{K3\pi}$**

- **$D \to K\pi\pi^0$: Large $R$ $\Rightarrow$ Significant coherence**

- **$D \to K3\pi$: $R \sim 0.3$, No significant coherence**
  - LHCb time dependent analysis $\Rightarrow$ Constrain $R$ and $\delta$
least squares fitter: used for extracting expected physics parameters from the correlated experimental data

Monte Carlo validation of the fitter

seven external inputs in the test: $R_{WS}$, $r^2$, $\delta_{K\pi}$, $x_D$, $y_D$, $x'^2$ and $y'$

their uncertainties are assumed to be uncorrelated

\[
R_{WS} = r^2 + ry_D \cos(\delta_{K\pi}) - rx_D \sin(\delta_{K\pi}) + \left(\frac{x_D^2 + y_D^2}{2}\right),
\]

\[
x' = x_D \cos\delta_{K\pi} + y_D \sin\delta_{K\pi},
\]

\[
y' = y_D \cos\delta_{K\pi} - x_D \sin\delta_{K\pi}.
\]
Sensitivity of the global fit at BESIII

- MC study corresponds to 3.0 / fb data
- input of the central values of the world average in 2012:
- with the external constraints of:
  \[ \delta_{K\pi} = 22.1^{+9.7}_{-11.1}(^\circ), \ y_D = 0.75 \pm 0.12(\%) \]

- output: \[ \delta_{K\pi} : \pm 8.3(\circ), \ y_D : \pm 0.10(\%) \]

- ~15% improvement on the parameters
- We expect large improvements on the strong phase and mixing parameters based on 20 /fb data.