Semileptonic D-decays at BESIII
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# On behalf of the BESIII collaboration 

$7^{\text {th }}$ Charm Meeting<br>May 18-22, 2015

## Outline

- Measurement Technique
- Study of $D^{+} \rightarrow K^{-} \pi^{+} e^{+} v_{e}$
- Study of $D^{+} \rightarrow \omega(\phi) e^{+} v_{e}$
- Study of $D^{+} \rightarrow K_{L} e^{+} v_{e}$


## Measurement Technique

- About $2.92 \mathrm{fb}^{-1}$ of data is collected at $\psi(3770)$, which ensures a pure $D \bar{D}$ final

- Branching fractions can be obtained using:

$$
\operatorname{Br}\left(D^{+} \rightarrow X e^{+} v_{e}\right)=\frac{\mathrm{N}_{\mathrm{sig}}}{\sum_{\alpha} \mathrm{N}_{\mathrm{tag}}^{\mathrm{obs}, \alpha} \epsilon_{\mathrm{tag}, \mathrm{sig}}^{\alpha} / \epsilon_{\mathrm{tag}}^{\alpha}}
$$

$N_{s i g}$ is the number of semileptonic candidates, $N_{\text {tag }}^{o b s, \alpha}$ the number of observed tagged mode $\alpha$, while $\epsilon_{\text {tag }}^{\alpha}$ and $\epsilon_{\text {tag,sig }}^{\alpha}$ the reconstruction efficiencies of tagged mode $\alpha$ and both the tagged and semileptonic mode.

- Six hadronic decay modes are chosen as tags:

$$
\begin{aligned}
& D^{+} \rightarrow \mathrm{K}^{-} \pi^{+} \pi^{+}, \mathrm{D}^{+} \rightarrow \mathrm{K}^{-} \pi^{+} \pi^{+} \pi^{0}, \mathrm{D}^{+} \rightarrow \mathrm{K}_{S}{ }^{0} \pi^{+} \\
& \mathrm{D}^{+} \rightarrow \mathrm{K}^{0}{ }^{0} \pi^{+} \pi^{0}, \mathrm{D}^{+} \rightarrow \mathrm{K}_{S}{ }^{0} \pi^{+} \pi^{+} \pi^{-}, \mathrm{D}^{+} \rightarrow \mathrm{K}^{+} \mathrm{K}^{-} \pi^{+}
\end{aligned}
$$

- Tags are selected based on two variables, and tag yield is obtained by fitting $m_{B C}$.

$$
\Delta E=E_{D}-E_{\text {beam }}, m_{B C}=\sqrt{E_{\text {beam }}^{2}-\left|\vec{p}_{D}\right|^{2}}
$$

## $m_{B C}$ Distribution

Tag yield is obtained by fitting $m_{B C}$. In the case of $D^{+} \rightarrow K^{-} \pi^{+} e^{+} v_{e}$ study, the fits are illustrated as below.







Signal: MC shape convoluting a double Gaussion; Background : Argus Function

## Study of $D^{+} \rightarrow K^{-} \pi^{+} e^{+} v_{e}$

In the $D^{+} \rightarrow K^{-} \pi^{+} e^{+} v_{e}$ decay, we can measure:

- Branching fractions
- The fractions and properties of different $K \pi$ (non-)resonant amplitudes
- S: non-resonant, $K_{0}^{*}(1430)$
- P: $K^{*}(892), K^{*}(1410)$
- D: $K_{2}^{*}(1430)$
- $q^{2}$ dependent transition form factors in $D^{+} \rightarrow \bar{K}^{* 0}(892) e^{+} v_{e}\left(q^{2}\right.$ is the invariant mass of $\left.e^{+} v_{e}\right)$
- The $D^{+} \rightarrow \bar{K}^{* 0}(892) e^{+} v_{e}$ decay can be described in terms of 3 helicity basis form factors: $\mathrm{H}_{ \pm, 0}\left(\mathrm{q}^{2}\right)$ (Any dependence on the lepton mass is neglected), which are measured in a model-independent way
- $\quad H_{ \pm, 0}\left(q^{2}\right)$ are generally written as linear combinations of a vector $\left(V\left(q^{2}\right)\right)$ and two axialvector $\left(A_{1,2}\left(q^{2}\right)\right)$ form factors, which are measured based on SPD (Spectroscopic Pole Dominance) model in the amplitude analysis

These measurements are important to compare with theoretical calculations and previous experiments

## Branching Fraction

- A nearly background-free ( $\sim 0.7 \%$ ) sample of more than 18000 candidates is selected. The $m_{K \pi}$ distribution is shown on the right.
- Branching fractions over the whole $m_{K \pi}$ range and in the $K^{* 0}(892)$ dominated window $[0.8,1] \mathrm{GeV} / c^{2}$ are calculated:

$$
\begin{array}{ll}
\operatorname{Br}\left(\mathrm{D}^{+} \rightarrow \mathrm{K}^{-} \pi^{+} \mathrm{e}^{+} v_{\mathrm{e}}\right) & =(3.71 \pm 0.03 \pm 0.09) \% \\
\operatorname{Br}\left(\mathrm{D}^{+} \rightarrow \mathrm{K}^{-} \pi^{+} \mathrm{e}^{+} \mathrm{v}_{\mathrm{e}}\right)_{[0.8,1]} & =(3.33 \pm 0.03 \pm 0.08) \%
\end{array}
$$



- Amplitude analysis is performed based on this sample (see next page).

The differential decay width of the $\mathrm{D}^{+} \rightarrow \mathrm{K}^{-} \pi^{+} \mathrm{e}^{+} v_{e}$ decay can be fully described using: [citation: N. Cabibbo and A. Maksymowicz, Phys. Rev. 137, B438 (1965)]

- $\mathrm{m}_{K \pi}$ - inv. mass squared of $K \pi$
- $q^{2}$ - inv. mass of $e^{+} v_{e}$
- $\theta_{K}, \theta_{e}, \chi$ angles



## Amplitude Analysis

## PDF Parameterization

(citation: BABAR Collaboration, Phys. Rev. D 83, 072001 (2011))

Unbinned Maximum likelihood fit (background considered)

- Non-resonant S-wave amplitude:

Magnitude: polynomial variation with $m_{K \pi}$ Phase $\delta_{S}$ : same as in LASS scattering experiment [Nucl. Phys. B296, 493 (1988)]

Other amplitudes: Breit-Wigner function with mass-dependent width

- Form factors are parameterized based on SPD model:

$$
V\left(q^{2}\right)=\frac{\mathrm{V}(0)}{1-q^{2} / m_{V}^{2}}, \quad A_{1,2}\left(q^{2}\right)=\frac{\mathrm{A}_{1,2}(0)}{1-q^{2} / m_{A}^{2}}
$$

Fit Results with $\mathbf{S + P}$ (preliminary)

- The fractions of the components can be extracted

$$
\begin{aligned}
f\left(\mathrm{D}^{+} \rightarrow\left(\mathrm{K}^{-} \pi^{+}\right)_{K^{* 0}(892)} \mathrm{e}^{+} v_{\mathrm{e}}\right) & =(93.93 \pm 0.22 \pm 0.18) \% \\
f\left(\mathrm{D}^{+} \rightarrow\left(\mathrm{K}^{-} \pi^{+}\right)_{S-\text { wave }} \mathrm{e}^{+} v_{\mathrm{e}}\right) & =(6.05 \pm 0.22 \pm 0.18) \%
\end{aligned}
$$

other components have significances less than $5 \sigma$ and correspond to fractions below $1 \%$

- The S-wave phase measured from amplitude analysis is illustrated in the following pages
- $m_{K^{* 0}(892)}=(894.60 \pm 0.25 \pm 0.08) \mathrm{MeV} / c^{2}$
$\Gamma_{K^{* 0}(892)}=(46.42 \pm 0.56 \pm 0.15) \mathrm{MeV} / c^{2}$
$r_{B W}=(3.07 \pm 0.26 \pm 0.11)(\mathrm{GeV} / c)^{-1}$
- $m_{V}=\left(1.81_{-0.17}^{+0.25} \pm 0.02\right) \mathrm{GeV} / c^{2}$ (first measurement)
$m_{A}=\left(2.61_{-0.17}^{+0.22} \pm 0.03\right) \mathrm{GeV} / c^{2}$
$A_{1}(0)=0.573 \pm 0.011 \pm 0.020$
$r_{V}=V(0) / A_{1}(0)=1.411 \pm 0.058 \pm 0.007$
$r_{2}=A_{2}(0) / A_{1}(0)=0.788 \pm 0.042 \pm 0.008$


## Projections of data and fitted MC distribution



The signal contains S-wave and $K^{* 0}(892)$ components.
In the lower histograms, $\chi$ of the (combined) bins of the upper histograms are provided.

## S-wave Phase Measurement

Instead of using the LASS parameterization for $\delta_{S}$, we fit the phase in different $m_{K \pi}$ intervals, assuming $\delta_{S}$ remains constant within each interval.

- Bin division: similar size for each bin, wider for the last two due to low population
- $\quad K^{* 0}(892)$ related parameters are also set free
- Blue dots: BESIII Model-independent measurement

Red or dotted lines: predicted by fit based on LASS parameterization

Green dots: BABAR Model-independent measurement
with $\mathrm{S}+\bar{K}^{* 0}(892)+\bar{K}^{* 0}(1410)$
[citation: BABAR Collaboration, Phys. Rev. D 83, 072001 (2011)]


Model-independent measurement of BESIII are consistent with its result from amplitude analysis within $1 \sigma$.

## Model-Independent Measurement of Form Factors

- Events located in the $K^{* 0}$ (892) window $[0.8,1] \mathrm{GeV} / \mathrm{c}^{2}$, are used to measure the form factors by a Projective Weighting Technique [citation: CLEO collaboration, Phys. Rev. D 81, 112001 (2010)].
- Signal is assumed to be composed of $K^{* 0}(892)$ and a non-resonant S-wave.
- Helicity basis form factors include:

P-wave related: $H_{ \pm, 0}\left(q^{2}\right)$
$S$-wave related: $h_{0}\left(q^{2}\right)$

- Five weighted $q^{2}$ histograms are built.

Weight is assigned to each event based on $\left(q^{2}, \cos \theta_{K}, \cos \theta_{e}\right)$.

- Form factors are independently computed in each $q^{2}$ bin.
- The model-independent measurements are generally consistent with CLEO's report and the predicted trend based on the SPD model from amplitude analysis.



## Study of $D^{+} \rightarrow \omega(\phi) e^{+} v_{e}$

- Current status of $D^{+} \rightarrow \omega(\phi) e^{+} v_{e}$

| $D^{+}$Decay Modes | Fraction | Confidence level |
| :--- | :---: | :---: |
| $D^{+} \rightarrow \omega e^{+} \nu_{e}$ | $(1.82 \pm 0.18 \pm 0.07) \times 10^{-3}$ |  |
| $D^{+} \rightarrow \phi e^{+} \nu_{e}$ | $<9.0 \times 10^{-5}$ | CL $=90 \%$ |

- Form factors of $D^{+} \rightarrow \omega e^{+} v_{e}$ have never been measured before
- No significant excess of $D^{+} \rightarrow \phi e^{+} v_{e}$ is observed
- $D^{+} \rightarrow \phi e^{+} v_{e}$ decay proceeds only through $\omega-\phi$ mixing or non-perturbative "Weak Annihilation" (WA) process (see Fig (b)). Measurement of its branching ratio can help to judge the dominant process.

(a) Feynman diagram representing the charged current process $D^{+} \rightarrow \omega e^{+} v_{e}$

(b) Feynman diagram representing the WA process $D^{+} \rightarrow \phi e^{+} v_{e}$


## Branching Fraction

- Semileptonic decays are identified using the variable $U$ :

$$
\begin{aligned}
& U=E_{\text {miss }}-\left|\vec{p}_{\text {miss }}\right|, \quad E_{\text {miss }}=E_{\text {beam }}-E_{\omega(\phi)}-E_{e} \\
& \vec{P}_{\text {miss }}=-\vec{P}_{\text {tag }}-\vec{P}_{\omega(\phi)}-\vec{P}_{e}, \quad \vec{P}_{\text {tag }}=\vec{P}_{\text {tag }} \sqrt{E_{\text {beam }}^{2}-m_{D}^{2}}
\end{aligned}
$$

- $U$ distribution for the $D^{+} \rightarrow \omega(\phi) e^{+} v_{e}$ decay:


Red dots: data
Black line: fit result
Blue area: total background
Green area: peaking background


Red dots: data
Black histogram: signal MC simulation
Arrows: signal region

- Branching fractions are compared with the world average value [citation: Particle Data Group, Chin. Phys. C, 527 38, 090001 (2014)].

| Mode | This work | Previous |
| :---: | :---: | :---: |
| $\omega e^{+} \nu_{e}$ | $(1.63 \pm 0.11 \pm 0.08) \times 10^{-3}$ | $(1.82 \pm 0.18 \pm 0.07) \times 10^{-3}$ |
| $\phi e^{+} \nu_{e}$ | $<1.3 \times 10^{-5}(@ 90 \%$ C.L. $)$ | $<9.0 \times 10^{-5}(@ 90 \%$ C.L. $)$ |

## Form Factors in $D^{+} \rightarrow \omega e^{+} v_{e}$

Form factors for $D^{+} \rightarrow \omega e^{+} v_{e}$ decay can be parameterized similarly as in the $D^{+} \rightarrow$ $K^{-} \pi^{+} e^{+} v_{e}$ decay. The projections and the form factor parameters are shown below:






Red dots: data Black Line: fit results Blue area: Background

$$
\begin{aligned}
& r_{V}=V(0) / A_{1}(0)=1.24 \pm 0.09 \pm 0.06 \\
& r_{2}=A_{2}(0) / A_{1}(0)=1.06 \pm 0.15 \pm 0.05
\end{aligned}
$$

## Study of $D^{+} \rightarrow K_{L} e^{+} v_{e}$ (first measurement)

- Branching fraction of $D^{+} \rightarrow K_{L} e^{+} v_{e}$ has never been measured before
- $K^{0}-\bar{K}^{0}$ mixing is expected to give rise to CP asymmetry with magnitude of about $-3.3 \times 10^{-3}$ in $D^{+} \rightarrow K_{L} e^{+} v_{e}$ decay [citation: Z.Z.Xing, Phys. Lett. B 353(1995)31; 363 (1995) 266]
- The differential decay width of $D^{+} \rightarrow K_{L} e^{+} v_{e}$ can be parameterized based on the transition form factor $f_{+}^{K}\left(q^{2}\right)$ and the CKM matrix element $\left|V_{c S}\right|$ :


$$
\frac{d \Gamma(D \rightarrow P e v)}{d q^{2}}=X \frac{G_{F}^{2}\left|V_{c s(d)}\right|^{2} p_{K(\pi)}^{3}}{24 \pi^{3}}\left|f_{+}\left(q^{2}\right)\right|^{2}
$$

For $P=K$ case, $X=1$

- Experimental study of $D^{+} \rightarrow K_{L} e^{+} v_{e}$ is important to test the theoretical prediction of $A_{C P}^{D^{+} \rightarrow K_{L} e^{+} v}{ }^{e}$, the LQCD calculation on $f_{+}^{K}(0)$ and the unitarity of the CKM matrix.


## Branching Fraction and $A_{C P}^{D^{+} \rightarrow K_{L} e^{+} v_{e}}$

- $K_{L}$ reconstruction:
- The direction of $K_{L}$ momentum can be determined from the induced shower in EMC.
- $K_{L}$ momentum can be inferred by constraining the neutrino $U=0$ (for $U$ definition see page 12).
- Because nuclear interaction is different for $K^{0}$ and $\bar{K}^{0}$, and $K^{0}-\bar{K}^{0}$ coherent oscillation is not considered in simulation, reconstruction efficiencies are corrected separately for $K_{L}$ from $K^{0}$ and $\bar{K}^{0}$
- Branching fraction:
- Signal yields are obtained by fitting $m_{B C}$ of the tag side (see next page).
- In this analysis, branching fraction is calculated separately for each charm and tag mode using:

$$
\mathcal{B}_{\text {sig }}=\frac{N_{\mathrm{DT}}\left(1-f_{\mathrm{bkg}}^{\text {peak }}\right)}{\epsilon N_{\mathrm{ST}}}
$$

- CP asymmetry is determined using:

$$
A_{C P} \equiv \frac{\mathcal{B}\left(D^{+} \rightarrow K_{L}^{0} e^{+} \nu_{e}\right)-\mathcal{B}\left(D^{-} \rightarrow K_{L}^{0} e^{-} \bar{\nu}_{e}\right)}{\mathcal{B}\left(D^{+} \rightarrow K_{L}^{0} e^{+} \nu_{e}\right)+\mathcal{B}\left(D^{-} \rightarrow K_{L}^{0} e^{-} \bar{\nu}_{e}\right)}
$$



| $D^{+} \rightarrow K_{L}^{0} e^{+} \nu_{e}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Tag Mode | $N_{\text {ST }}$ | $N_{\text {DT }}$ | $f_{\text {bkg }}^{\text {peak }}$ (\%) | $\epsilon(\%)$ | $\mathcal{B}_{\text {sig }}(\%)$ |
| $D^{-} \rightarrow K^{+} \pi^{-} \pi^{-}$ | $410200 \pm 670$ | $10492 \pm 103$ | $41.83 \pm 0.28$ | $33.96 \pm 0.10$ | $4.381 \pm 0.050$ |
| $D^{-} \rightarrow K^{+} \pi^{-} \pi^{-} \pi^{0}$ | $120060 \pm 457$ | $3324 \pm 64$ | $44.78 \pm 0.49$ | $33.14 \pm 0.19$ | $4.613 \pm 0.103$ |
| $D^{-} \rightarrow K_{S}^{0} \pi^{-} \pi^{0}$ | $102136 \pm 378$ | $2658 \pm 56$ | $38.93 \pm 0.58$ | $35.67 \pm 0.21$ | $4.456 \pm 0.108$ |
| $D^{-} \rightarrow K_{S}^{0} \pi^{-} \pi^{-} \pi^{+}$ | $59158 \pm 303$ | $1459 \pm 41$ | $40.84 \pm 0.76$ | $32.51 \pm 0.27$ | $4.488 \pm 0.145$ |
| $D^{-} \rightarrow K_{S}^{0} \pi^{-}$ | $47921 \pm 225$ | $1287 \pm 36$ | $38.90 \pm 0.88$ | $35.07 \pm 0.32$ | $4.679 \pm 0.155$ |
| $D^{-} \rightarrow K^{+} K^{-} \pi^{-}$ | $35349 \pm 239$ | $905 \pm 32$ | $44.64 \pm 0.97$ | $30.98 \pm 0.35$ | $4.575 \pm 0.190$ |
| Averaged |  |  |  |  | $4.455 \pm 0.038$ |
| $D^{-} \rightarrow K_{L}^{0} e^{-} \bar{\nu}_{e}$ |  |  |  |  |  |
| Tag Mode | $N_{\text {ST }}$ | $N_{\text {DT }}$ | $f_{\text {bkg }}^{\text {peak }}$ (\%) | $\epsilon(\%)$ | $\mathcal{B}_{\text {sig }}(\%)$ |
| $D^{+} \rightarrow K^{-} \pi^{+} \pi^{+}$ | $407666 \pm 668$ | $10354 \pm 103$ | $40.44 \pm 0.29$ | $34.02 \pm 0.11$ | $4.447 \pm 0.051$ |
| $D^{+} \rightarrow K^{-} \pi^{+} \pi^{+} \pi^{0}$ | $117555 \pm 450$ | $3264 \pm 63$ | $42.28 \pm 0.52$ | $33.19 \pm 0.19$ | $4.829 \pm 0.107$ |
| $D^{+} \rightarrow K_{S}^{0} \pi^{+} \pi^{0}$ | $101824 \pm 378$ | $2642 \pm 55$ | $39.06 \pm 0.58$ | $35.92 \pm 0.21$ | $4.402 \pm 0.104$ |
| $D^{+} \rightarrow K_{S}^{0} \pi^{+} \pi^{+} \pi^{-}$ | $59046 \pm 303$ | $1533 \pm 42$ | $39.68 \pm 0.77$ | $33.44 \pm 0.27$ | $4.683 \pm 0.147$ |
| $D^{+} \rightarrow K_{S}^{0} \pi^{+}$ | $48240 \pm 226$ | $1217 \pm 35$ | $38.50 \pm 0.88$ | $35.20 \pm 0.32$ | $4.408 \pm 0.147$ |
| ${ }^{++} \rightarrow K^{+} K^{-} \pi^{+}$ | $35742 \pm 240$ | $942 \pm 32$ | $44.04 \pm 0.95$ | $32.40 \pm 0.36$ | $4.552 \pm 0.181$ |
| Averaged |  |  |  |  | $4.508 \pm 0.038$ |

## Branching Fraction and $A_{C P}^{D^{+} \rightarrow K_{L} e^{+} v_{e}}$

The fraction of peaking backgrounds are estimated by MC.

Black dots: data;
Blue: Fit result;
Green Line: combinatorial background


Branching fraction:
$\bar{B}\left(D^{+} \rightarrow \boldsymbol{K}_{L} e^{+} v_{e}\right)=(4.482 \pm 0.027 \pm 0.103) \%$
CP asymmetry:
$A_{C P}^{D^{+} \rightarrow K_{L} e^{+} v_{e}}=(-0.59 \pm 0.60 \pm \mathbf{1 . 5 0}) \%$



$M_{\mathrm{BC}}\left(\mathrm{GeV} / \mathrm{C}^{2}\right)$

$M_{\mathrm{BC}}\left(\mathrm{GeV} / \mathrm{C}^{2}\right)$


$M_{\mathrm{BC}}\left(\mathrm{GeV} / \mathrm{C}^{2}\right)$

## Form Factor measurement

Signal shape of $q^{2}$ distribution can be described using $\frac{d n_{\text {observed }}}{d q^{2}}=A N_{\text {tag }} p^{3}\left(q^{\prime 2}\right)\left|f_{+}\left(q^{\prime 2}\right)\right|^{2} \epsilon\left(q^{\prime 2}\right) \otimes \sigma\left(q^{\prime 2}, q^{2}\right)$ $\underset{\text { [cite: Becher and Hill, Phys. Lett. B } 633,61 \text { (2006) }}{\text { 2-par. Series Expansion is performed form factor } f_{+}\left(q^{2}\right): \quad f_{+}\left(q^{2}\right)=\frac{1}{P\left(q^{2}\right) \phi\left(q^{2}, t_{0}\right)} \sum_{k=0}^{\infty} a_{k}\left(t_{0}\right)\left[z\left(q^{2}, t_{0}\right)\right]^{k}}$

Simultaneous fits are performed:






$f_{+}^{K}(0)\left|V_{c s}\right|=0.728 \pm 0.006 \pm 0.011, \quad r_{1} \equiv a_{1} / a_{0}=1.91 \pm 0.33 \pm 0.24$

## Summary

- In the study of $D^{+} \rightarrow K^{-} \pi^{+} e^{+} v_{e}$ :
- Branching fractions are measured:

$$
\begin{array}{ll}
\operatorname{Br}\left(D^{+} \rightarrow K^{-} \pi^{+} e^{+} v_{e}\right) & =(3.71 \pm 0.03 \pm 0.09) \% \\
\operatorname{Br}\left(D^{+} \rightarrow K^{-} \pi^{+} e^{+} v_{e}\right)_{[0.8,1]} & =(3.33 \pm 0.03 \pm 0.08) \%
\end{array}
$$

- Amplitude analysis is applied:
- Fractions of the $K \pi$ components are analyzed. S-wave contribution is observed to be ( $6.05 \pm 0.22 \pm 0.18$ ) $\%$.
- $K^{* 0}(892)$ properties and the form factors based on the SPD model are provided.
- Model-independent measurement of S-wave phase and the $K^{* 0}(892)$ helicity basis form factors are performed. They are generally consistent with previous reports and the amplitude analysis results.
- In the study of $D^{+} \rightarrow \omega(\phi) e^{+} v_{e}$ :
- Branching fractions or upper limits are provided:

$$
\begin{aligned}
& \operatorname{Br}\left(D^{+} \rightarrow \omega e^{+} v_{e}\right)=(1.63 \pm 0.11 \pm 0.08) \times 10^{-3} \\
& \operatorname{Br}\left(D^{+} \rightarrow \phi e^{+} v_{e}\right)<1.3 \times 10^{-5}(@ 90 \% \text { C. L. })
\end{aligned}
$$

- Form factor parameters in $\mathrm{D}^{+} \rightarrow \omega \mathrm{e}^{+} v_{\mathrm{e}}$ are first measured:

$$
r_{V}=V(0) / A_{1}(0)=1.24 \pm 0.09 \pm 0.06 ; \quad r_{2}=A_{2}(0) / A_{1}(0)=1.06 \pm 0.15 \pm 0.05
$$

- In the study of $D^{+} \rightarrow K_{L} e^{+} v_{e}$ :
- Branching fractions and CP assymetry are measured:

$$
\bar{B}\left(D^{+} \rightarrow K_{L} e^{+} v_{e}\right)=(4.482 \pm 0.027 \pm 0.103) \% \cdot A_{C P}^{D^{+} \rightarrow K_{L} e^{+} v_{e}}=(-0.59 \pm 0.60 \pm 1.50)
$$

- Form factor related parameters are also measured:

$$
f_{+}^{K}(0)\left|V_{C S}\right|=0.728 \pm 0.006 \pm 0.011, r_{1} \equiv a_{1} / a_{0}=1.91 \pm 0.33 \pm 0.24
$$

## Backup

## Estimation of Backgrounds in the Double Tag

By using MC-truth information of the $K_{L}$ efficiency corrected $D \bar{D}$ MC samples, the double-tag $D$ candidates can be divided into the following categories:
$>$ Signal: tag-side matched and signal-side matched signal events
> Background:

- Bkg I: $D \bar{D}$ decays of which hadronic tag $D$ is misreconstructed and non- $D \bar{D}$ processes. Its proportion varies from $1 \%$ to $\mathbf{1 2 \%}$ according to the specific hadronic tag mode
- Bkg II: $(\sim 10 \%) D^{+} \rightarrow K_{L} e^{+} v_{e}$ events of which $K_{L}$ shower is mis-reconstructed.
- Bkg III: $D^{+} \rightarrow X_{e} v_{e}$ non-signal events ( $\sim 24 \%$ ), which are from $D^{+} \rightarrow \bar{K}^{*}(892)^{0} e^{+} v_{\boldsymbol{e}}(41.9 \%), D^{+} \rightarrow K_{S} e^{+} v_{e}$ $(41.2 \%), D^{+} \rightarrow \pi^{0} e^{+} v_{e}(10.2 \%), D^{+} \rightarrow \eta e^{+} v_{e}(6.0 \%)$ and $D^{+} \rightarrow \boldsymbol{\omega} \boldsymbol{e}^{+} \boldsymbol{v}_{\boldsymbol{e}}(\mathbf{0 . 7 \%})$
- Bkg IV: $D^{+} \rightarrow X \mu v_{\mu}$ events ( $\sim 3 \%$ ), consist of $D^{+} \rightarrow$ $K_{L} \boldsymbol{\mu}^{+} \boldsymbol{v}_{\boldsymbol{\mu}}(\mathbf{6 5 . 2 \%}), D^{+} \rightarrow \bar{K}^{*}(\mathbf{8 9 2})^{0} \boldsymbol{\mu}^{+} \boldsymbol{v}_{\boldsymbol{\mu}}(\mathbf{2 3 . 3 \%})$ and $D^{+} \rightarrow K_{S} \mu^{+} v_{\mu}(11.5 \%)$

Composition of double-tag $\boldsymbol{D}$ candidates

$q^{2}\left(\mathrm{GeV}^{2} / c^{4}\right)$

Bkg V: Non-leptonic $D$ decay events ( $\sim \mathbf{3 \%}$ ), which are from $D^{+} \rightarrow \bar{K}^{0} \boldsymbol{\pi}^{+} \boldsymbol{\pi}^{\mathbf{0}} \mathbf{( 7 8 \% )}$ and $D^{+} \rightarrow \bar{K}^{0} K^{*}(892)^{+}(22 \%)$
In the determination of $B\left(D^{+} \rightarrow K_{L} e^{+} v_{e}\right)$, the peaking backgrounds consist of Bkg II $\sim$ Bkg V.
This estimation brings in $\mathbf{1 . 6 \%}$ systematic uncertainty.

